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Dr. J. H. JEANS, PRESIDENT, in the Chair.

P. R. Chidambava Aiyar, Assistant at the Kodaikánal Observatory, S. India, and

Robert Barker, "Brendon," Crossbrook Street, Cheshunt, Herts,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Thomas Davies, B.A., B.Sc., 82 Warley Road, Langley, near Birmingham (proposed by J. G. Howarth); and

Rev. James William St. Clare Hill, M.A., Principal of the Royal School for the Blind, Leatherhead, Surrey (proposed by T. E. R. Phillips).

One hundred and thirty-three presents were announced as having been received since the last Ordinary Meeting, including, amongst others:—

Amsterdam, Astronomical Institute, Publications No. 1: A. Pannekoek, Researches on the Structure of the Universe.

Tycho Brahe: Opera Omnia, tomus xi. (presented by Dr. Dreyer).

Royal Observatory, Greenwich: First Greenwich Catalogue of Stars for 1925.

Optical Society, Transactions: Vol. 26, No. 1.

F. C. Molesworth: The Military Uses of Astronomy.

W. H. Steavenson: Catalogue of Instruments made and/or used by Sir William Herschel (presented by Lady Lubbock).

On the Masses, Luminosities, and Surface-Temperatures of the Stars. (Second Paper.) By J. H. Jeans.

19. In the first paper we obtained the equation

$$q\frac{\partial\lambda}{\partial q}=\frac{Kq^s}{\lambda}-(\lambda+1) \quad . \quad . \quad . \quad . \quad (62)$$

where q is the pressure of radiation at any point inside a star, λ is the ratio of gas-pressure to pressure of radiation, and K and s are constants. The substitution

$$Kq^s=x^2$$

transforms this equation into

$$\tfrac{1}{2}sx\lambda\frac{\partial\lambda}{\partial x}=x^2-\lambda(\lambda+1) \quad . \quad . \quad . \quad . \quad (63)$$

Assume a solution in ascending powers of s in the form

$$\lambda=\Lambda_0+s\Lambda_1+s^2\Lambda_2+ \ldots . \quad . \quad . \quad . \quad (64)$$

Substituting this value for λ into equation (63), and equating the coefficients of the different powers of s, we obtain

$$x^2-\Lambda_0(\Lambda_0+1)=0 \quad . \quad . \quad . \quad . \quad (65)$$

$$\tfrac{1}{2}x\Lambda_0\frac{\partial\Lambda_0}{\partial x}=-\Lambda_1(2\Lambda_0+1) \quad . \quad . \quad . \quad . \quad (66)$$

$$\tfrac{1}{2}x\frac{\partial}{\partial x}(\Lambda_0\Lambda_1)=-\Lambda_1{}^2+\Lambda_2(2\Lambda_0+1) \quad . \quad . \quad . \quad (67)$$

Equation (65) gives at once

$$\Lambda_0=(x^2+\tfrac{1}{4})^{\frac{1}{2}}-\tfrac{1}{2} \quad . \quad . \quad . \quad . \quad (68)$$

whence from equations (66) and 67)

$$\Lambda_1=-\frac{x^2}{4x^2+1}\Lambda_0$$

$$\Lambda_2=-\left[\frac{x^4+x^2}{(4x^2+1)^2}+\frac{x^4-x^2}{2(4x^2+1)^{\frac{5}{2}}}\right]\Lambda_0 \quad . \quad . \quad . \quad (69)$$

The solution is rapidly convergent for small values of s. The values of s with which we shall finally be concerned are those around $s=\frac{1}{18}$. For such values of s we may neglect terms in s^2, and adopt as the solution of equation (63),

$$\lambda=\Lambda_0+s\Lambda_1=\Lambda_0\left(1-\frac{sx^2}{4x^2+1}\right) \quad . \quad . \quad . \quad (70)$$

Let us further put $\lambda=x\psi$ and $\Lambda_0=x\Psi_0$, so that

$$\psi=\left(1-\frac{sx^2}{4x^2+1}\right)\Psi_0 \quad . \quad . \quad . \quad . \quad (71)$$

The following table gives computed values of Λ_0, Ψ_0, and $1-\frac{sx^2}{4x^2+1}$ in its first four columns, the last quantity being calculated for the value $s=\frac{1}{18}$.

TABLE I.

x.	Λ_0.	Ψ_0.	$1-\frac{sx^2}{4x^2+1}$.	λ.	$1-\beta$.	M.
100	99·50	0·995	0·986	98·12	·0101	0·415
50	49·50	0·990	0·986	48·81	·0201	0·599
40	39·50	0·988	0·986	38·95	·0250	0·674
30	29·50	0·983	0·986	29·09	·0332	0·790
20	19·51	0·975	0·986	19·23	·0494	0·997
15	14·51	0·967	0·986	14·31	·0653	1·186
12	11·51	0·959	0·986	11·35	·0810	1·365
10	9·512	0·951	0·986	9·376	·0963	1·539
9	8·514	0·946	0·986	8·392	·1063	1·651
8	7·516	0·940	0·986	7·409	·1189	1·800
7	6·518	0·931	0·986	6·425	·1346	1·986
6	5·521	0·920	0·986	5·442	·1553	2·24
5·5	5·023	0·913	0·986	4·951	·1681	2·40
5	4·525	0·905	0·986	4·461	·1831	2·60
4·5	4·028	0·895	0·986	3·971	·201	2·85
4	3·531	0·883	0·986	3·481	·223	3·22
3·5	3·035	0·867	0·986	2·992	·251	3·61
3	2·541	0·847	0·986	2·505	·285	4·24
2·8	2·344	0·837	0·987	2·311	·302	4·58
2·6	2·148	0·826	0·987	2·118	·321	4·95
2·4	1·951	0·813	0·987	1·923	·342	5·47
2·2	1·756	0·798	0·987	1·731	·366	6·07
2	1·562	0·781	0·987	1·540	·394	6·91
1·8	1·368	0·760	0·987	1·348	·426	8·00
1·6	1·176	0·735	0·987	1·160	·463	9·55
1·5	1·081	0·721	0·987	1·066	·483	10·5
1·4	0·987	0·705	0·988	0·974	·507	11·9
1·3	0·893	0·687	0·988	0·881	·532	13·4
1·2	0·800	0·667	0·988	0·791	·559	15·5
1·1	0·708	0·644	0·988	0·699	·588	18·2
1·0	0·618	0·618	0·989	0·611	·621	22·1

We notice that throughout the range from $x=\infty$ to $x=1$ the values of $\left(1-\frac{sx^2}{4x^2+1}\right)$ differ from one another by only about three parts in a thousand. As this is less than the observational errors in the quantities we shall ultimately be concerned with, we may legitimately treat

the quantity in question as constant, assigning to it the value $1-\frac{1}{4}s$, which it assumes when $x=\infty$. This represents the factor $\left(\frac{2}{s+2}\right)^{\frac{1}{2}}$ which occurred throughout our previous work. We accordingly replace equation (70) by

$$\lambda=\left(\frac{2}{s+2}\right)^{\frac{1}{2}}\Lambda_0=\left(\frac{2}{s+2}\right)^{\frac{1}{2}}x\Psi_0 \quad . \quad . \quad . \quad . \quad (72)$$

20. Since $p=q\lambda$, we have

$$q\lambda=\frac{\mathrm{R}\rho\mathrm{T}}{m\mu}=\frac{\mathrm{R}\rho}{m\mu}\left(\frac{q}{\frac{1}{3}a}\right)^{\frac{1}{3}},$$

giving

$$\rho=\mathrm{A}q^{\frac{1}{4}}\lambda \quad . \quad . \quad . \quad . \quad . \quad (73)$$

where

$$\mathrm{A}=\frac{m\mu}{\mathrm{R}}(\tfrac{1}{3}a)^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad . \quad (74)$$

From the equations

$$\frac{d}{dr}(p+q)=-\frac{\gamma\rho}{r^2}\int_0^r 4\pi\rho r^2 dr$$

$$\frac{d}{dr}(p+q)=\frac{\mathrm{K}q^s}{\lambda}\frac{dq}{dr}$$

of §§ 3 and 4 we obtain

$$\frac{r^2}{\gamma\rho}\frac{\mathrm{K}q^s}{\lambda}\frac{dq}{dr}=-\int_0^r 4\pi\rho r^2 dr,$$

which gives, on differentiation with respect to r,

$$\frac{1}{4\pi r^2}\frac{d}{dr}\left[\frac{\mathrm{K}}{\gamma}\frac{r^2q^s}{\rho\lambda}\frac{dq}{dr}\right]+\rho=0.$$

Since, however,

$$\frac{\mathrm{K}q^s}{\rho\lambda}=\frac{x^2}{\mathrm{A}q^{\frac{1}{4}}\lambda^2}=\frac{1}{\mathrm{A}\psi^3q^{\frac{1}{4}}},$$

the equation becomes

$$\frac{1}{4\pi\gamma r^2}\frac{d}{dr}\left[\frac{r^2}{\mathrm{A}\psi^2}q^{-\frac{1}{4}}\frac{dq}{dr}\right]+\rho=0 \quad . \quad . \quad . \quad . \quad (75)$$

From equation (73)

$$\rho=\mathrm{A}q^{\frac{1}{4}}\lambda=\mathrm{A}q^{\frac{1}{4}}x\psi=\mathrm{A}\left(\frac{x^2}{\mathrm{K}}\right)^{\frac{3}{4s}}x\psi,$$

so that the equation further reduces to

$$\frac{1}{\pi\gamma r^2}\frac{d}{dr}\left[\frac{r^2}{\mathrm{A}\psi^2}\frac{d}{dr}\left(\frac{x^2}{\mathrm{K}}\right)^{\frac{1}{4s}}\right]+\mathrm{A}\psi x\left(\frac{x^2}{\mathrm{K}}\right)^{\frac{3}{4s}}=0 \quad . \quad . \quad (76)$$

We are proceeding on the supposition that s is small. If s were actually zero, x, or $(\mathrm{K}q^s)^{\frac{1}{2}}$, would be constant throughout the star, so that λ and ψ would be constant throughout. In this case the whole solution would reduce to that given by Eddington's analysis.

Now in equation (76) the terms $\left(\frac{x^2}{K}\right)^{\frac{1}{4s}}$ and $\left(\frac{x^2}{K}\right)^{\frac{3}{4s}}$ vary rapidly on account of the smallness of s, while ψ and A only vary slightly. We shall accordingly obtain a good approximation by treating ψ and A as constants in equation (76), assigning to ψ the value as given by Eddington's analysis for the star under discussion.

Thus, if we write

$$R^2=r^2A^2\psi^3\pi\gamma K^{-\frac{1}{2s}},$$

in which A and ψ are treated as constants, equation (76) reduces to

$$\frac{1}{R^2}\frac{d}{dR}\left[R^2\frac{d}{dR}x^{\frac{1}{2s}}\right]+x^{1+\frac{3}{2s}}=0;$$

or, with $x=y^{2s}$,

$$\frac{1}{R^2}\frac{d}{dR}\left[R^2\frac{dy}{dR}\right]+y^{2s+3}=0 \quad . \quad . \quad . \quad . \quad (77)$$

which is of Emden's standard form.

If R_1, y_1 refer to any special solution of this equation, the most general solution is

$$y=fy_1, \quad R=f^{-(s+1)}R_1 \quad . \quad . \quad . \quad . \quad . \quad (78)$$

where f is a constant.

The mass M of a star is given by

$$M=-\frac{r^2}{\gamma\rho}\frac{d}{dr}(p+q)=-\frac{r^2}{\gamma\rho}\frac{Kq^s}{\lambda}\frac{dq}{dr},$$

or, after substitution and simplification,

$$M=\frac{4f^{-s}}{\gamma A^2(\pi\gamma)^{\frac{1}{2}}\psi^{\frac{1}{2}}}R_1^2\frac{dy_1}{dR_1}.$$

Since $A \propto \mu$, we obtain from this

$$\log M=-s\log f-2\log\mu-\tfrac{7}{2}\log\psi+\text{cons.} \quad . \quad . \quad (79)$$

Again, we have, as in § 12, at the surface of the star,

$$\tfrac{1}{3}aT^4=q_0=\frac{E}{8\pi Cr^2}=\frac{E}{8\pi CR_1^2}A^2\psi^3\pi\gamma K^{-\frac{1}{2s}}f^{2s+2},$$

whence, using the relation $K=M/E \times$ *a constant*, we obtain

$$4\log T=\left(1+\frac{1}{2s}\right)\log E+2\log\mu+3\log\psi-\frac{1}{2s}\log M+(2s+2)\log f+\text{cons.} \quad (80)$$

The elimination of f between this and equation (79) leads to

$$(2s+2\tfrac{1}{2})\log M+4s\log T-(s+\tfrac{1}{2})\log E$$
$$+(2s+4)\log\mu+(4s+7)\log\psi=\text{cons.} \quad . \quad (81)$$

which agrees with our former equation (53) when μ and ψ are regarded as constants.

21. If for the moment we neglect the small variations in μ and, as before, put

$$\log E = -0{\cdot}4m + \text{cons.},$$

where m is the absolute magnitude of the star, equation (81) assumes the form

$$(2s+2\tfrac{1}{2})\log M + 4s \log T + \tfrac{1}{5}(2s+1)m + (4s+7)\log\psi = \text{cons.} \quad . \quad (82)$$

It does not appear possible to progress further without assuming a definite value for s. Determining s from the six stars used in the previous paper, it appears that the best fit is obtained by taking $s=\frac{1}{18}$. In this case equation (82) assumes the form*

$$m + \log T_e + 11\tfrac{3}{4}\log M + 32\tfrac{1}{2}\log \Psi_0 = 8{\cdot}332 \quad . \quad . \quad (83)$$

the constant on the right being the average value for the six stars. The agreement of this equation with observation is shown in the following table. All the observational material is taken from Eddington's table, except that I have used Sampson's value 12800° for the effective temperature of Sirius.

TABLE II.

Star.	T_e.	M.	Ψ.	$m_{obs.}$	$m_{calc.}$	$\Delta M/M$. (O−C).
Sun	5860°	1·00	0·975	4·89	4·92	−·006
α Centauri *b*	5000	1·14	0·967	4·53	4·45	+·016
α Centauri *f*	3700	0·97	0·978	5·24	5·24	−·000
Capella *b*	5200	4·18	0·847	−0·36	−0·34	−·004
Capella *f*	7400	3·32	0·875	0·22	0·23	−·001
Sirius	12800	2·45	0·910	0·97	0·99	−·003

The last column shows the change in the assumed mass which would be necessary to satisfy equation (83) exactly. It will be seen that all the errors are well within those of observation.

The mean numerical value of $\Delta M/M$ for the six stars used is $\pm$·005. I have carried out similar calculations for the value $s=\frac{1}{16}$ and $\frac{1}{20}$, and find that in each of these cases the mean value of $\Delta M/M$ is $\pm$·007; with $s=\frac{1}{10}$ this mean value is $\pm$·019. Thus $s=\frac{1}{18}$ appears to give a true solution of minimum error for the data as given in our table, although it is clear that the errors of adjacent solutions would still be within the limits of errors of observation.

With a view to testing equation (83) further afield, I have checked it by using the observational material, as used by Eddington, for Shapley's eclipsing variables, limiting the test to those which Shapley indicates as first-grade orbits. The result is shown in the following table :—

* Here the T and ψ of equation (82) have been replaced by T_e and Ψ_0, to which they are proportional.

TABLE III.

Star.	T_e.	M.	Ψ.	$m_{obs.}$	$m_{calc.}$	$\Delta M/M$.
	°					
V Puppis	19000	19·2	0·64	−4·61	−4·66	+0·01
u Herculis	17000	7·6	0·76	−2·89	−2·34	−0·11
U Ophiuchi	11500	5·36	0·815	−0·47	−1·30	+0·17
β Aurigæ	10500	2·38	0·911	0·13	1·21	−0·22
W Urs. Maj.	6200	0·74	0·985	5·42	6·29	−0·17

According to Sampson's determinations, the temperatures assumed for the first three stars are substantially too low, but this can hardly account for much of the error. The largest error in the whole table is that for β Aurigæ, which happens to be a fairly close copy of Sirius as regards mass and spectral type. As our equation gave accurate results for Sirius, it seems permissible to attribute the error in β Aurigæ, and so also that in the rest of the table, to errors of observation. The alternative supposition, that stars of similar mass and spectral type can differ widely in respects not considered by our analysis, would be difficult to reconcile with the closeness of fit obtained in Table II.

22. The quantity s for which we have obtained the value $s=\frac{1}{18}$ is not quite identical with the quantity s which appeared in the former paper. For in the present paper we have assumed μ, the mean molecular weight, to be constant throughout the star, whereas in the previous paper we assumed it to vary from point to point of the star, being connected with the density ρ by the relation

$$\frac{\mu}{\mu_0}=\left(\frac{\rho}{\rho_0}\right)^{1-\phi} \quad . \quad . \quad . \quad . \quad . \quad (84)$$

where μ_0, ρ_0 are the values of μ and ρ at the centre of the star.

If we write s_0 for the value assumed by s when μ is treated as constant ($\phi=1$), the result of the discussion of the present paper is expressed by the equation

$$s_0=\tfrac{1}{18}.$$

For a star whose mass is not very great (*cf.* § 13),

$$\frac{2s+4}{2s+3}\phi=n=\frac{2s_0+4}{2s_0+3},$$

so that, on putting $s_0=\frac{1}{18}$, we must have

$$\frac{2s+4}{2s+3}\phi=\frac{37}{28},$$

from which it is of course impossible to determine s and ϕ separately. The value $s=\frac{1}{8}$ would require $\phi=1{\cdot}011$, which is probably too small; $s=\frac{3}{8}$ gives $\phi=1{\cdot}043$, which may be too large.

If we are taking account of variations of molecular weight, it should be noticed that equation (84), which we have assumed to be true throughout any one star, will not in general be true from one star to another. To make it valid throughout the whole of each star and also

from one star to another, we must allow either μ_0 or ρ_0 to vary from one star to another, and this reintroduces the term $(2s+4)\log\mu$ in equation (81). To keep the right-hand member of equation (84) strictly constant, this term ought now to be replaced by

$$(2s+4)[\log\mu_0-(1-\phi)\log\rho_0].$$

It hardly seems worth examining the effects of the variation of μ further, in view of the fact that the errors involved in our present analysis appear to be already within those of observation.

23. The relation

$$m+\log T_e+11\tfrac{3}{4}\log M+32\tfrac{1}{2}\log\Psi_0=8\cdot332$$

which we have obtained between the absolute magnitude, effective temperature, and mass of a star can be exhibited graphically in a variety of ways. Probably the most convenient is to take m and $\log T_e$ as ordinate and abscissa respectively, so that (since Ψ_0 is a function of M only) the curves M=*constant* are a series of parallel lines. Such a diagram is shown in fig. 1, the horizontal scale for log T being ten times the vertical scale for m.

The slant lines represent the masses indicated ; these meet the axis T=25000° or log T=4·398 at the points

m=7·579, 4·291, 2·551, 1·421, 0·604, −0·028, −0·930, −1·565, −2·060, −2·470, −2·803, −3·104, and −3·333 respectively.

24. The primary object of the present investigation is to examine what evolutionary consequences, if any, can be deduced from the present arrangement of the stars. The parallel dotted lines in the diagram opposite mark approximately the limits of the stars shown in Russell's well-known "compass-diagram," on which the giant-dwarf theory was originally built ; the dotted line starting from the point (approximately) m=1, Sp=Fo marks approximately the lower limit of the giant stars in Russell's diagram.

Additional evidence as to the distribution of giant stars is provided by Shapley's studies of star-clusters. The chain-lines in the region bounded by m=+1 to m=−3 and Sp=B5−Mo represent approximately the distribution found by Shapley in M3 ; * other star-clusters tell substantially the same story.

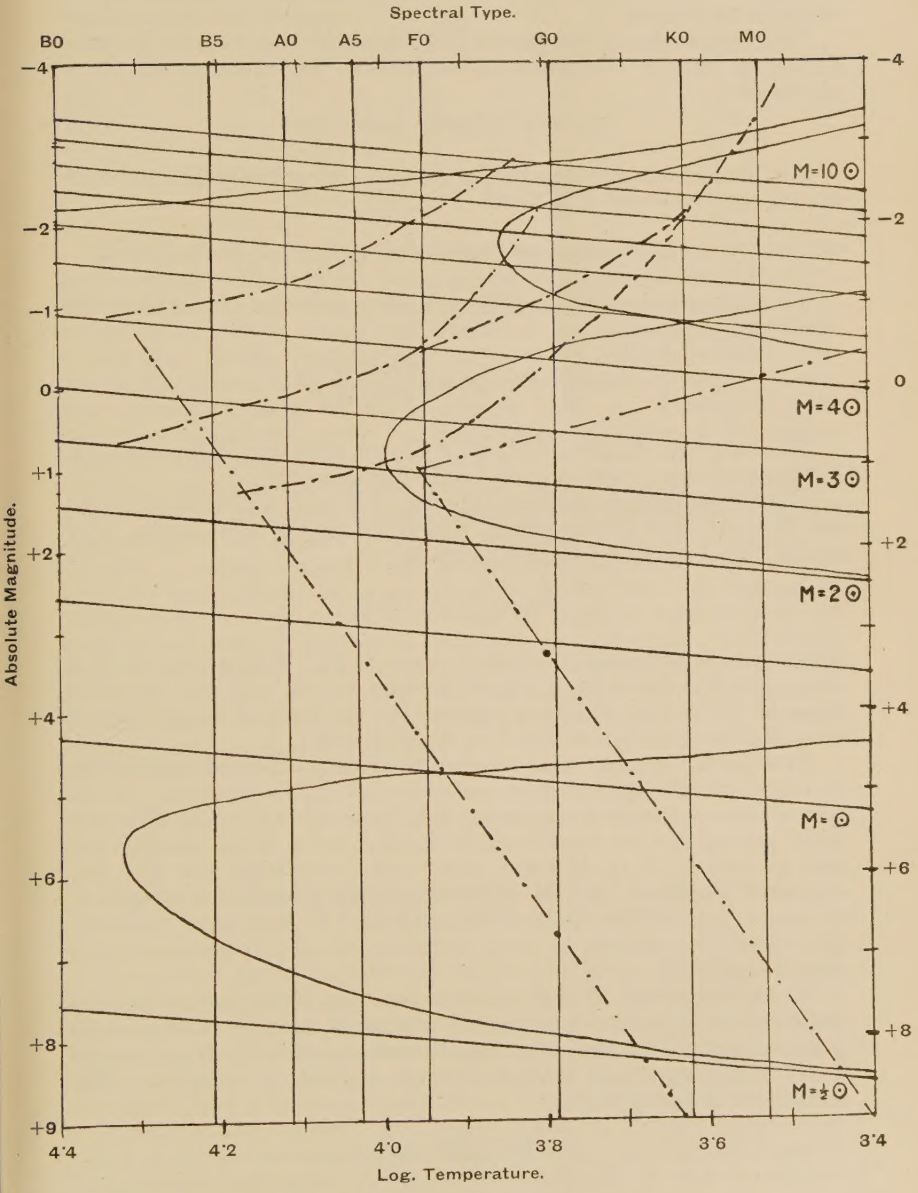
It is significant that these last three lines are all approximately parallel to the line which marks Russell's limit to the giant stars.

It would appear to be exceedingly difficult to explain these distributions on the supposition that the stars remained of constant mass, and we turn at once to discuss the possibility of a decrease of mass.

25. The simplest case of decreasing mass occurs when the stars consist of uniform substance which decreases with a uniform modulus of decay κ. If M_0 is the initial mass of a star, its mass after time t is $M_0e^{-\kappa t}$. The emission at the same time is

$$E=-\frac{1}{C^2}\frac{dM}{dt}=\frac{\kappa}{C^2}M,$$

* *Mount Wilson Contribution* 155, p. 2 ; or *Astroph. Journ.*, **49** (1919), 96.


Spectral Type.
B0
B5
A0
A5
F0
G0
K0
M0
−4
−2
−1
0
+1
+2
+4
+6
+8
+9
M=10⊙
M=4⊙
M=3⊙
M=2⊙
M=⊙
M=½⊙
Absolute Magnitude.
4·4
4·2
4·0
3·8
3·6
3·4
Log. Temperature.

so that E/M retains the constant value κ/C^2 throughout the whole evolution of the star.

The disposition of the curves E/M=cons. in the diagram is comparatively simple. In terms of m and M the equations of these curves are

$$m+2{\cdot}5 \log M=\text{cons.} \quad . \quad . \quad . \quad . \quad (85)$$

Let us momentarily denote the abscissa and ordinate of a point in this diagram by x and y, so that

$$dy=-dm\ ;\ dx=-10d(\log T).$$

From equations (85) and (82) we readily find that the slope of the curves (85) is given by

$$dx=\left[s+2+(4s+7)\frac{d\log\Psi_0}{d\log M}\right]\frac{dy}{s}.$$

The value of $d(\log\Psi_0)/d(\log M)$ is always negative, being equal to $-{\cdot}0052$ at $x=50$, $-{\cdot}016$ at $x=2{\cdot}5$, and $-{\cdot}21$ at $x=50$.

Clearly the value of dy/dx is a function of M only, so that any straight line M=cons. cuts all the curves E/M=cons. at the same angle. For $x=1{\cdot}25$ (M=14·4 ⊙) the slope is given by

$$sdx=(0{\cdot}53+0{\cdot}16)dy,$$

which gives dy/dx equal to about $\frac{1}{10}$ when $s=\frac{1}{18}$. With $s=\frac{1}{10}$, which is still a possible value for s, dy/dx is equal to ·18, and this is nearly equal to the slope of the lines representing the limit of Russell's giant stars and the distribution found by Shapley in M3.

Thus, so far as giant stars are concerned, their observed distribution is consistent with an evolution under the law E/M=constant. As we pass to lower values of M the curves E/M become less steep, so that they curve precisely in the way in which the M3 curves in our diagram are seen to curve. It is, however, clear that, no matter how well the supposed constancy of E/M may explain the distribution of giants, it cannot account for that of the dwarfs. If E/M were constant, the surface-temperature of a star ought continually to increase as its mass diminished.

26. As suggested by fig. 2 in the first paper, we proceed to the consideration of a star consisting of a mass M_0 of inert matter which is not capable of annihilation, together with a mass $M-M_0$ of matter which annihilates itself with a constant modulus of decay κ. The emission E is now given by

$$E=\frac{\kappa}{C^2}(M-M_0),$$

so that the evolutionary paths in the diagram will be the curves

$$m+2{\cdot}5 \log (M-M_0)=\text{cons.} \quad . \quad . \quad . \quad . \quad (86)$$

The three curves of somewhat parabolic shape in the diagram represent the three curves

$$m+2{\cdot}5\log\left(\frac{M}{\odot}-4\right) \quad =-0{\cdot}75,$$

$$m+2{\cdot}5\log\left(\frac{M}{\odot}-1{\cdot}883\right)=0{\cdot}475,$$

$$m+2{\cdot}5\log\left(\frac{M}{\odot}-0{\cdot}5\right) \quad =4{\cdot}5.$$

These and all the curves included in equation (86) are of somewhat similar shape. Their upper parts, corresponding to values of M which are large in comparison with M_0, approximate to the curves E/M=cons. considered in the last section; their lower parts are of course asymptotic to the lines $M=M_0$. Any of the curves could undergo a bodily displacement parallel to the axis of T without substantial change of shape. The curves are drawn for $s=\frac{1}{18}$; a larger value of s would give more open curves, and *vice versa*. In the limiting case of $s=0$, the curves reduce to lines parallel to the axis of T.

The important point is that for any positive value of s the curves exhibit the bending back to lower values of T which is essential for the interpretation of Russell's "compass-diagram" as an evolutionary diagram. Almost certainly our model star, consisting of a mixture of only two kinds of matter, is too simple to fit the facts. A mixture of many types, as suggested in § 18, would fit the facts better and would seem in any case to represent a more natural occurrence. I have not thought it necessary to draw curves for more complicated cases, but it is, I think, clear that a mixture of substances of varying degrees of destructibility would lead to an evolutionary curve of the general type required by Russell's diagram.

On the Mass-Luminosity Relation; a reply to Dr. Jeans.

By A. S. Eddington, M.A., M.Sc., F.R.S.

In a recent paper * I have reached the conclusion that the absolute bolometric magnitude of a star depends mainly on its mass, so that after applying a small correction depending on effective temperature the stars should lie on a particular curve in a magnitude-mass diagram. Such observational data as we possess seem to confirm the curve I calculated, and I have regarded this as an indication that the theory is in main outline correct. Dr Jeans † considers that the results have a very different interpretation. It is desirable that I should reply to his criticism and state my reasons for adhering to the conclusions already expressed.

At the outset Jeans states that "when the problem is treated in a

* *M.N.*, **84**, 308. † *M.N.*, **85**, 196.

sufficiently general way the rather surprising properties which Professor Eddington attributes to the stars are found to disappear." His generalisation consists in removing "the special assumptions which underlie Professor Eddington's investigations of the same problem"—in particular the law according to which the absorption-coefficient is supposed to depend on density and temperature. It seems to me natural that when this law is discarded the results should disappear; it would have been rather disappointing if the long study of the physical theory of absorption had contributed nothing essential to the problem. There seems, however, to be a suggestion in the paper that a modification of the law, trifling from a physical point of view, would be sufficient to lead to widely different results; but if so, there is nothing in the mathematical analysis to bear it out.

The results of Jeans' more general assumptions can be reached at once without his long analysis. The fundamental formulæ of the theory are

$$d(p+\tfrac{1}{3}aT^4)=\frac{4\pi cGM}{L}\cdot\frac{1}{\eta k}d(\tfrac{1}{3}aT^4) \quad . \quad . \quad . \quad (1)$$

$$d(p+\tfrac{1}{3}aT^4)=-g\rho dr \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where p is the gas-pressure, $\frac{1}{3}aT^4$ the radiation-pressure, M the mass, L the energy radiated per second, k the absorption-coefficient, c the velocity of light, and G the constant of gravitation; η is unity at the boundary and varies inwards in a way determined by the distribution of the source of the star's energy.

Jeans takes k to be proportional to ρ/T^{3+n}, and (within any one star) η proportional to T^l. The approximation used in my papers corresponds to $n-l=0$.

The principal point at issue is the dependence of the star's luminosity on effective temperature. This is easily found since, so long as the mass of the star is unaltered, the solutions of (1) and (2) are homologous. If, keeping the mass of every element fixed, we multiply its linear dimensions by R, the following factors must be introduced:—T is multiplied by R^{-1}, ρ by R^{-3}, p and $\frac{1}{3}aT^4$ by R^{-4}, g by R^{-2}, k by R^n, L by R^{-n}, η is unaltered. The equations then continue to be satisfied. Hence

$$L \propto R^{-n} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where R is the radius. But in terms of effective temperature T_e

$$L=4\pi R^2 \cdot \tfrac{1}{4}acT_e^4.$$

Hence, eliminating R,

$$L \propto T_e^{4n/(2+n)}.$$

Converting into absolute magnitude m, we have

$$-\delta m=\frac{10n}{2+n}\delta(\log T_e) \quad . \quad . \quad . \quad . \quad (4)$$

For my law of absorption $n=\frac{1}{2}$, and we obtain the correction already given in my previous paper,* $\delta m=-2\delta(\log T_e)$. To take n greatly

* *Loc. cit.*, p. 310.

different from $\frac{1}{2}$ would be to reject altogether the law of absorption on which I have based my conclusions ; but, of course, I do not suppose the law $k \propto \rho/T^{\frac{7}{2}}$ to be exact, and I have nothing against values of n reasonably near to $\frac{1}{2}$, say between 0 and 1. Equation (4) shows that there is nothing singular about $n=\frac{1}{2}$, and that if the law is approximately true the results will be approximately true. The result that stars of different effective temperature are confined to a narrow band in the magnitude-mass diagram is unaffected by reasonable changes of n. The range from type M to type A, say 3000°–12000°, gives $\delta(\log T_e)=0{\cdot}6$; hence by (4) the spread in absolute magnitude is $1^{m}{\cdot}2$ for $n=\frac{1}{2}$ and $2^{m}{\cdot}0$ for $n=1$.

In referring to my magnitude-mass curve, Jeans says (§ 18) : " The observed values of L and M for actual stars ought, according to Eddington, to lie on, or close to, this curve," and he proceeds to claim that " our wider theory " upsets this conclusion as soon as $n-l$ differs infinitesimally from zero. " In fact the mass-luminosity law disappears entirely." It will be seen that this claim is entirely at variance with the mathematical results.

In § 12 Dr. Jeans revives an old criticism of his to which I have replied more than once : " Thus mathematical analysis brings us back to the common-sense view that a star of mass M, generating energy at a given rate E, can always adjust its radius and surface-temperature so as to be in equilibrium." Unfortunately mathematical analysis shows plainly that the common-sense view will not work. For if the given rate of generation of energy is greater than the rate of radiation L fixed by the formula, the energy of the star is increasing and it must expand. By (3) L varies as R^{-n}, and therefore diminishes.* This augments the discrepancy. Accordingly the star fails to find equilibrium and goes on expanding indefinitely.

Since we find actual stars in equilibrium, we must take the view (which is perhaps not entirely opposed to common sense) that a star does not generate energy at a *given* rate, but at a rate which it can modify by expanding or contracting. It is this flexibility which enables it to find a state of equilibrium, the small variation of L with R tending rather to hinder the process.

We shall now obtain briefly and more specifically the remaining results of Jeans' paper in order to show that nothing deadly to the theory is concealed in the long investigation. He works out only two cases, (*a*) stars of very large mass in which p is neglected in comparison with $\frac{1}{3}aT^4$, (*b*) stars of very small mass in which $\frac{1}{3}aT^4$ is neglected in comparison with p.

(*a*) In this case (1) becomes

$$d(\tfrac{1}{3}aT^4)=\frac{4\pi cGM}{L}\cdot\frac{1}{\eta k}\,d(\tfrac{1}{3}aT^4). \quad . \quad . \quad . \quad (5a)$$

so that

$$L=4\pi cGM/\eta k \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

* Jeans admits that n is positive—" $s[=\frac{1}{4}(n-l)]$ is almost certainly positive in actual fact " (*loc. cit.*, p. 208).

Since (in the same star) $\eta k \propto \rho/T^{3+n-l}$, this gives

$$\rho \propto T^{3+n-l},$$

and for this case the whole pressure P is $\frac{1}{3}aT^4$, so that

$$P \propto \rho^{4/(3+n-l)} \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

Accordingly the distribution is of the well-known polytropic type $P \propto \rho^\gamma$.

Comparing two stars at homologous points (viz. the centres) so that η is the same for both, we have by (6*a*)

$$L \propto M/k \propto MT^{3+n}/\rho \propto R^3T^{3+n}.$$

But the central pressure in homologous stars varies as M^2/R^4, so that $T \propto M^{\frac{1}{2}}/R$. Hence

$$L \propto M^{\frac{1}{2}(3+n)}R^{-n} \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

or, eliminating R by the relation $L \propto R^2T_e^4$,

$$\log L = \frac{3+n}{2+n}\log M + \frac{4n}{2+n}\log T_e + \text{const.} \quad . \quad . \quad (9a)$$

(*b*) Equation (1) becomes

$$dp = \frac{4\pi cGM}{L} \cdot \frac{1}{\eta k} d(\tfrac{1}{3}aT^4) \quad . \quad . \quad . \quad . \quad (5b)$$

Hence in the same star

$$\rho dp \propto T^{3+n-l}d(T^4),$$

or, since $p \propto \rho T$,

$$pdp \propto T^{7+n-l}dT.$$

Hence

$$p \propto T^{4+\frac{1}{2}(n-l)}, \quad \rho \propto T^{3+\frac{1}{2}(n-l)} \quad . \quad . \quad . \quad (7b)$$

and the distribution is again of the type $p \propto \rho^\gamma$.

Comparing two different stars at the centre we have, on account of the homology,

$$\frac{p}{T^4} \propto \frac{dp}{d(T^4)} \propto \frac{M}{Lk} \propto \frac{MT^{3+n}}{L\rho}$$

by (5*b*). As before, $p \propto M^2/R^4$, $\rho \propto M/R^3$, so that $T \propto M/R$. Hence we obtain

$$L \propto M^{5+n}R^{-n} \quad . \quad . \quad . \quad . \quad . \quad (8b)$$

and finally

$$\log L = \frac{2(5+n)}{2+n}\log M + \frac{4n}{2+n}\log T_e + \text{const.} \quad . \quad . \quad (9b)$$

Equations (7*a*) and (7*b*) are equivalent to Jeans' formulæ (38) and (36); but (9*a*) and (9*b*) appear to disagree with his final results (56). His results seem to be contrary to the general formula (4) of this paper, but I cannot altogether understand the notation. Equation (54), apparently determining the law of change of molecular weight

in terms of n and l, is altogether mysterious, unless Dr. Jeans has used n with two different meanings in § 14.

It will be seen that there is nothing singular about my value $n=\frac{1}{2}$. Jeans apparently no longer attaches much importance to introducing the exponent l, since (rather unnecessarily) he drops it before the end of his work. It will be seen that it does not affect the magnitude-mass-temperature relation in the two cases considered by him.

In § 2 he states that his investigation will treat the ratio of gas-pressure to radiation-pressure, λ, as a variable quantity, in contrast to my investigation in which λ is constant within the star. His paper can scarcely be regarded as fulfilling this promise. After carrying the work a little way he gives it up at equation (40) without reaching any mass-luminosity relation, and falls back on cases (*a*) and (*b*). So far as results bearing on the title of his paper are concerned, his generalisation consists in limiting λ to the two values 0 and ∞. The complicated formulæ which he reaches before abandoning the task are a consequence of using an inconvenient expression for η, and the essential difficulties of the problem (which are considerable) do not begin until after passing this point. There is no special virtue in the form $\eta \propto T^l$, which has no direct physical interpretation; it must not be confused with a law expressing the rate of generation of energy as a function of the temperature. We can, however, see that a positive value of l represents a concentration of the source of energy to the centre (which is presumably an improvement) and introduces a sink of energy in the outer parts (which is presumably harmful). Whether the balance is to the good or the bad is doubtful; I should suspect that small values of l will make some improvement, but that if it is attempted to represent a strong concentration in this way more harm than good will be done. An alternative form $\eta \propto T^{3+l}/\rho$ has precisely the same general effect (both advantageous and disadvantageous), and avoids analytical difficulties. There seems no reason to prefer the more difficult form, since it does not give any closer representation of the physical problem. With $\eta \propto T^{3+l}/\rho$ and $k \propto \rho/T^{3+n}$, (1) becomes

$$d(p+\tfrac{1}{3}aT^4)=\text{const.}\times T^{n-l}d(\tfrac{1}{3}aT^4),$$

so that

$$p+\tfrac{1}{3}aT^4=\text{const.}\times(\tfrac{1}{3}aT^4)^{1+s},$$

where $s=\frac{1}{4}(n-l)$. Or, writing q for the radiation-pressure,

$$p=q(Aq^s-1).$$

The corresponding investigation by Jeans stops at this point. Further development would be easier with this relation than with Jeans' infinite series; but I do not feel much interest in the continuation, since in another paper in this number I have given the full numerical solution for a form which is free from the drawbacks mentioned above.

A Limiting Case in the Theory of Radiative Equilibrium.

By A. S. Eddington, M.A., M.Sc., F.R.S.

1. This paper deals with the radiation of heat from a star supplied with energy by a *point-source at the centre*. It is desirable to explain first why this ideal case is important.

In the equations for the equilibrium of a star there appears a function expressing the relative distribution of the source of energy in different parts of the star. This distribution is still unknown. Progress with the theory became possible when it was realised that the results were very insensitive to comparatively large changes in the adopted distribution. The quantity that appears in the equations is not the generation of energy ϵ at the point considered, but the average generation η throughout the mass interior to the point considered—a much more slowly varying function of r. For example, if ϵ is proportional to the temperature and therefore has a range of about 1000 : 1 between the centre and the outside, the range of η is only 1·7 : 1.

The general effect of changes in the distribution of the source can be seen in the following way. The temperature-gradient in a star achieves two purposes : it gives a pressure-gradient great enough to extend the mass M of the star against gravity, and it drives the radiation L (that is to be emitted) through the star against the obstructing opacity. The latter task is made easier by distributing the source away from the centre ; part of L will then have a shortened journey to the surface and less temperature-gradient will be required in the deeper parts of the star. Thus L/M is increased by spreading the source of energy, and decreased by concentrating it near the centre. The law of distribution of the source concerns us because it fixes the average (optical) depth of the source below the surface ; but a precise evaluation of this depth will not be required until the theory reaches a high degree of refinement, since there is no difficulty in making an estimate near enough for a first approximation.

2. A special reason for relegating to the background the possible corrections due to this uncertainty lay in the fact that η occurs in the equations only in combination with the absorption-coefficient k as a product ηk. It was idle to consider a small change of η so long as much larger changes of k were probable. There is now, however, physical ground for expecting that k will vary as $\rho/T^{\frac{7}{2}}$ (subject to a small correction appreciable in the least massive stars). We here accept this law—not necessarily as a proved result, but because it is necessary to compare it with the results of astronomical observation.

If $k \propto \rho/T^{\frac{7}{2}}$ and $\epsilon \propto T$ an interesting result appears.* Then ηk is constant to a degree of accuracy beyond all reasonable requirements, and the approximation used in my former solution is justified beyond expectation. Dividing the star into 10 shells of equal mass, the relative

* *M.N.*, **83**, 40.

values of ηk at the centre and at the boundaries of successive shells are found to be

1·70, 1·68, 1·67, 1·65, 1·67, 1·69, 1·72, 1·76, 1·84, 2·02.

In the outermost shell ηk continues to increase slowly; just before the boundary it would run up sharply to about 20, but the region where this failure occurs is of negligible mass and it is unnecessary to deal with it in the solution. For convenience of reference I shall call the distribution $\epsilon \propto T$ the "usual distribution," since it is the one for which my usual method gives the most trustworthy results.

Of course there is no reason to expect this special law to hold. For comparison we give the corresponding figures for $\epsilon \propto T^4$—a much higher concentration of the source of energy. These are

4·71, 3·60, 3·14, 2·81, 2·55, 2·33, 2·17, 2·06, 2·02, 2·10.

The approximation ηk=const. is inferior, but is still reasonable for a first approximation.

The absolute values of these figures represent $(\eta/\eta_0)(k/k_c)$, η_0 being the value at the boundary (η_0=L/M) and k_c the central value. Denoting the mean values of the two sets of figures, viz. 1·74 and 2·75, by α, we have by the usual theory

$$4\pi c G(1-\beta)=\eta k=\alpha\eta_0 k_c=\alpha k_c L/M,$$

so that

$$L=\frac{4\pi c GM(1-\beta)}{\alpha k_c}.$$

If the mass and radius are kept constant and the law of ϵ is varied, β and k_c are unaltered (so long as the approximation is admissible), and hence L varies inversely as α. The change from $\epsilon \propto T$ to $\epsilon \propto T^4$ will thus reduce the brightness of the star in the ratio 1·74 : 2·75 or about $0^m\cdot5$.

3. I have found * on applying Kramers' theory of absorption to stellar conditions that the calculated absorption is about $\frac{1}{8}$ of that required by astronomical observation. The *law* of absorption is confirmed satisfactorily, but the absolute amount is discrepant. A correction indicated by Rosseland † increases the discrepancy to about $\frac{1}{12}$; his paper has the even more important result of greatly narrowing the field in which possible physical explanations of the discrepancy may be looked for. Does the discrepancy arise on the astronomical or the physical side of the investigation?

It is for the purpose of making the astronomical side more secure in regard to this comparison that I have taken up again the problem of the distribution of the source. To agree with observation we want to reduce by nearly 3 magnitudes the calculated brightness of a star of given mass, and we have seen that concentrating the source towards the centre helps in this direction. The question arises, What is the utmost decrease of brightness that can be effected? It is scarcely likely to be large enough for our object, but we cannot draw any certain

* *M.N.*, **84**, 104. † *M.N.*, **84**, 525.

conclusions from approximate methods of treatment. To find the utmost limit I have considered the energy to be entirely concentrated at the centre—a single point-source—and have made the necessary numerical calculations for a typical star. The effect turns out to be surprisingly small, considering that this is an extreme departure from the conditions contemplated in the theory.

4. The differential equations of the theory are

$$dp_R = -k\rho H dr/c \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$dP = -g\rho dr \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here P is the whole pressure made up of the gas-pressure $p_G = \Re\rho T/m$ and the radiation-pressure $p_R = \frac{1}{3}aT^4$. In the present problem

$$H = L/4\pi r^2, \quad k = k_1\rho/T^{\frac{7}{2}},$$

where L is the energy radiated per second by the star and therefore the energy required to be emitted by the point-source. Equation (1) thus becomes

$$dp_R = -\frac{k_1 L}{4\pi c} \cdot \frac{\rho^2}{r^2 T^{\frac{7}{2}}}\, dr \quad . \quad . \quad . \quad . \quad (3)$$

With (2) and (3) we can carry on a quadrature. After each step ρ and T are found from P and p_R; the mass of the shell is computed and added to the current total, and hence the new value of g is found. The usual guessing of half-way values and other devices of quadratures are employed. The serious trouble arises with regard to starting values. Ordinarily we identify the star considered by its mass M and radius R; but these are only found at the end of the calculation. In their place we must have two disposable initial conditions. One of these is the value of $k_1 L$; this amounts to settling at the start the bolometric magnitude of the star, since k_1 is merely a physical constant of the material. The other condition can be taken as fixing the value of either T or ρ at some point near the centre. (The density at the centre is zero, the mass being driven away by intense radiation-pressure; so we start at some fixed distance from the centre where the density is still very low and the interior mass is only just beginning to be of account.) Unfortunately *both* T and ρ must be fixed before we can start the quadratures, so that we overcondition the problem and usually fail to obtain any solution; what actually happens is that P reaches zero before p_R, or *vice versa*, thus violating the boundary condition that both pressures approximately vanish simultaneously. Accordingly we keep ρ fixed and try successively different values of T, until at last we close down on the value which makes P and p_R reach zero together. Of course a very great amount of calculation is wasted on the unsuccessful trials.

5. The calculation is so long that we do not care to undertake it for more than one mass. It is therefore important to choose initial conditions to give a star in the most interesting part of the mass-range. I wished to obtain a star something like Capella, and actually hit it off fairly well. The following values were taken,

$$\log_{10} k_1 L = 62{\cdot}6590, \qquad m = 2{\cdot}2 \ (\log_{10} \Re/m = 7{\cdot}5744),$$

and the solution began at $r=0{\cdot}9 \cdot 10^{11}$ cm. with an adopted density 0·01. After many trials two initial temperatures were found giving solutions which kept close together through the main mass of the star for one of which p_G vanished first and for the other p_R. These accordingly defined narrow limits between which the actual condition of the star must lie. The two solutions gave for the mass M_r up to the sphere $r=8{\cdot}1 \cdot 10^{11}$ cm. the values $9{\cdot}687 \cdot 10^{33}$ and $9{\cdot}706 \cdot 10^{33}$, so that the limits were close enough to fix the mass with ample accuracy.

Finally, as a complete check I interpolated suitable * starting values of P, p_R, and M_r at $r=8{\cdot}1 \cdot 10^{11}$ and worked backwards to the centre, at the same time doubling the number of intermediate steps. Going in this direction the mass M_r is stripped off in successive shells and the test of the solution is that M_r shall just reach zero at $r=0$. The test was well fulfilled; it was possible to go past the original starting-point to $r=0{\cdot}5 \cdot 10^{11}$, and the small mass $\cdot 0020 \cdot 10^{33}$ then remaining would suffice to fill the interior with a mean density ·0038, which is (as nearly as we can judge) in the proper relation to the actual density ·0065 at the point reached. There were 76 steps in the final solution, and the results at every fourth step are given in the table.

Numerical Solution.

r.	M_r.	ρ.	T.	p_R.	p_G.
·5	·0020	·652	6·93	5·889	1·697
·9	·0234	·989	6·68	5·065	2·477
1·3	·0941	1·273	6·43	4·344	3·070
·7	·2536	1·501	6·16	3·675	3·473
2·1	·5435	1·659	5·88	3·043	3·660
2·5	·997	1·726	5·57	2·452	3·609
2·9	1·628	1·697	5·24	1·916	3·336
3·3	2·422	1·578	4·88	1·448	2·892
3·7	3·337	1·391	4·51	1·058	2·356
4·1	4·313	1·166	4·14	·7489	1·811
4·5	5·286	·934	3·77	·5144	1·321
4·9	6·198	·719	3·41	·3441	·9202
5·3	7·012	·535	3·07	·2250	·6162
5·7	7·707	·388	2·74	·1443	·3990
6·1	8·278	·274	2·44	·0909	·2511
6·5	8·734	·1895	2·17	·05641	·1543
6·9	9·088	·1288	1·92	·03448	·09268
7·3	9·356	·0861	1·69	·02071	·05456
7·7	9·554	·0567	1·48	·01215	·03146
8·1	9·699	·0372	1·28	·00675	·01780
Unit 10^{11}	10^{33}	10^{-2}	10^{6}	10^{12}	10^{12}

* Suitable for carrying the solution much further towards the boundary before either P or p_R runs off to zero.

It is evident from the trend of the column M_r that not much mass remains to be added after the last line. Approximate methods suffice to compute the small amount to come; its weight has to balance p_R+p_G, and we know more or less the average value of r where it lies. The correction is found to be $\Delta M=0{\cdot}26\,.\,10^{33}$ gm. The additional radius (which is not required with any great accuracy) is similarly found to be $\Delta R=3{\cdot}25\,.\,10^{11}$ cm.

6. The result, then, is that a star for which

$$M=9{\cdot}96\,.\,10^{33}\text{ gm.}=5{\cdot}02\times\odot,\quad R=11{\cdot}35\,.\,10^{11}\text{ cm.}$$

emits radiation at the rate given by

$$\log k_1 L=62{\cdot}6590.$$

For comparison we calculate by the ordinary formulæ the radiation L′ for a star with the same M and R but with the "usual" distribution of the source of energy ($\epsilon\propto T$). This gives

$$\log k_1 L'=63{\cdot}0467,$$

so that

$$\log (L'/L)=0{\cdot}388,$$

corresponding to a magnitude-difference $0^m{\cdot}97$.

In my work on the absorption-coefficient I have already discounted to some extent this effect of concentration of the source by adopting 2·5 instead of 1·74 for the factor a. The utmost further diminution of brightness is thus half a magnitude, and the discrepancy of the absorption-coefficient cannot be accounted for by any change in the assumed distribution of the source of energy.

7. Examination of equations (2) and (3) shows that if all lengths are altered in the ratio [R] and L is altered in the ratio $[R]^{-1}$, the masses being unchanged, the solution remains homologous. Hence for stars of the same mass $L\propto R^{-1}$. This result is the same as for the "usual" distribution, so that the magnitude-difference 0·97 is the same whatever the radius of the star. It applies, however, solely to stars of mass $5{\cdot}02\times\odot$. For our main purpose this limitation is harmless, since the discrepancy of the absorption-coefficient is conspicuous for all masses, and, moreover, we have attached special importance to the discrepancy for Capella, whose mass $4{\cdot}2\times\odot$ is near the value here treated.

To test the effect for stars of smaller mass would require another lengthy computation. But it is fairly evident that it will be of roughly the same magnitude or perhaps rather smaller. By taking a rather massive star, in which radiation-pressure and gas-pressure are well balanced, we have given the variation of ηk the best chance of introducing a large correction; but it has not risen to the opportunity. I conclude that the case here treated is typical of the stars in general up to mass 5, and that the uncertainty in our determinations of brightness due to ignorance of the distribution of sub-atomic energy does not exceed $\pm 0^m{\cdot}5$. This is the extreme, not the probable, uncertainty.

8. Attention may be called to a few points of interest in the table. The ratio of p_R to p_G in the outer part settles down to a value near

0·37 ; for the usual distribution the ratio is 0·52 throughout. A concentration of the source to the centre is thus likely to help the formation of the more massive stars, since the radiation-pressure is less strong in the outer parts where other perturbing forces such as centrifugal force would be most threatening to the stability.

In the central $\frac{1}{10}$ of the mass the density decreases inwards. It is doubtful if such a condition could actually occur, and it is, of course, physically absurd to postulate a source of sub-atomic energy in a region of zero density. But the solution is not intended to represent an actual star ; it is the limit on one side beyond which the conditions cannot pass and which they may not be able to reach.

In the usual distribution, at points where $M_r/M=\frac{1}{8}, \frac{7}{8}$ the temperatures are in the ratio 2·8 : 1 ; for the point-source the ratio is 2·5 : 1. The ratios of density are respectively 22·5 : 1 and 9·3 : 1. The point-source thus leads to slightly more uniform temperature and considerably more uniform density in the main part of the star.

The Origin of the Solar System, in relation to Professor Eddington's Theory of Stellar Luminosity. By Harold Jeffreys, M.A., D.Sc.

The theory of the constitution and radiation of stars that has recently been developed by Professor Eddington leads to inferences concerning the former state of the sun that have an important bearing on certain aspects of the tidal theory of the origin of the solar system.

In the first place, we have sufficiently definite information about the age of the earth from other sources. The ratios of the amounts of uranium and lead in terrestrial rocks lead to the result that the age of the earth is greater than $1\cdot3\times10^9$ years, and less than 8×10^9 years. The theory of the reduction of the eccentricities of the orbits of the planets by a gaseous resisting medium leads to an estimate of from 10^9 to 10^{10} years for the age of Mercury. The theory of tidal friction indicates an age of the moon of order, 4×10^9 years. Now on the tidal theory of the origin of the solar system the ages of the planets are all practically equal, and the moon is not more than a few thousand years younger than the earth. The surface temperature of the earth has been substantially the same throughout geological time, and therefore the rate of solar radiation may be supposed constant throughout the same period.

The present rate of solar radiation amounts to $3\cdot3\times10^{33}$ ergs per second, corresponding to a loss of mass of $3\cdot7\times10^{12}$ grams per second, or $1\cdot17\times10^{20}$ grams per year. Thus the loss of mass in 10^{10} years, which has been seen to be an outside estimate of the age of the solar system, would be a little over 10^{30} grams, or about 1/2000 of the mass of the sun. Even if solar radiation was greater in pre-geological time than now, it seems inconceivable that it can have been so great as to

invalidate the conclusion that the mass of the sun has been practically the same ever since the planets were formed.

To put the matter in a more definite form, we have the equation *

$$1-\beta=0{\cdot}00309M^2\mu^4\beta^4 \quad . \quad . \quad . \quad . \quad (1)$$

where μ is the mean molecular weight, which Eddington takes to be 2·11, M is the ratio of the mass to the present mass of the sun, and β is a number less than unity. So long as M is between 0·5 and 2, we have with sufficient accuracy

$$1-\beta=0{\cdot}05M^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The luminosity L is proportional to $r^2T_e^4$, where r is the radius and T_e the effective temperature; it is also proportional to $M^5(1-\beta)^3\mu^7T_e^{\frac{1}{2}}$ by Eddington's equation (3). Using (2) we therefore have for a star of small mass, with μ constant,

$$L \propto M^{\frac{22}{5}}T_e^{\frac{1}{2}} \propto r^2T_e^4 \quad . \quad . \quad . \quad . \quad (3)$$

whence

$$T_e \propto M^{\frac{11}{8}}r^{-\frac{5}{8}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$L \propto M^{\frac{11}{2}}r^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But

$$L \propto \frac{dM}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Now the presumption is that r decreases with the time. Thus we shall overestimate the rate of loss of mass in the past if we treat r as constant in (5). Then (5) and (6) together give a differential equation for M, whose solution is

$$1-M^{-\frac{9}{2}}=\tfrac{9}{2}t\left[\frac{dM}{dt}\right]_{t=0} \quad . \quad . \quad . \quad . \quad (7)$$

It immediately follows that, with the actual value of the present rate of loss of mass by radiation, the mass 10^{10} years ago cannot have exceeded the present mass by one part in 2000.

The first considerable departure of these results from those of previous work on the origin of the solar system is indicated by equation (5). In my book, *The Earth*, I based several inferences on the giant and dwarf theory of stellar evolution. According to this theory the sun is now a dwarf star, and must, when it was gaseous, have radiated a great deal more rapidly than at present. In order that it should have behaved as a star at all, on the other hand, it appeared necessary that its effective temperature should have been at least 3100°. Information then available suggested that a gaseous star of the mass of the sun must have had a radius not greater than $2{\cdot}1\times10^7$ km. to have such a temperature; if its radius were smaller its temperature would be higher. The supposition that the solar system was formed while the sun was a gaseous star therefore fixed an upper limit of $2{\cdot}1\times10^7$ km. to the radius of the sun at the time of the tidal encounter.

* Eddington, *M.N.R.A.S.*, **84**, 309, 1924.

On the present theory the conditions are quite different. We see from (5) that the more the sun was distended the less would be its luminosity. Again, we see from (4) that, subject to the approximate constancy of M, r must vary as T_e^{-5}. Further, the ratio can be fixed by reference to the sun at present, with r equal to $6{\cdot}96\times10^5$ km. and T_e equal to 5860°. The result of the comparison is that when T_e was 3100°, r must have been $1{\cdot}93\times10^6$ km. Using (5), we find that the luminosity must have had 0·60 of its present value, and the density 0·047 of its present value. It must be remarked, however, that the variation of luminosity with radius depends on the law of absorption of radiation adopted, and is less definitely indicated than the dependence on mass.

These estimates are to be regarded as the extremes permissible. Strictly speaking, there is no need to suppose that the sun at the time of the encounter was any more distended than at present. It appears probable from pp. 185–7 of Jeans's *Problems of Cosmogony* that in a slow tidal encounter rupture would take place by emission from a point on the surface, so long as the distribution of density corresponded to a ratio of specific heats less than 2·2. The analogy with disruption by rotation suggests, indeed, that the critical ratio of specific heats is still greater. Now on Eddington's theory the distribution of density in a gaseous star is the same as for a gas whose ratio of specific heats * is $\frac{4}{3}$, subject to a slight increase if the mean molecular weight increases from the centre to the outside. Thus the disruption of the primitive sun in a slow encounter would take place by the formation of a filament. This would hold whatever the mean density may have been, so long as the star is gaseous, which on Eddington's theory is true even at the present time. There seems to be no reason to think that in the actual encounter the result would differ appreciably, except in scale, from that appropriate to masses of much smaller mean density. †

If the sun is contracting on account of loss of mass, it would be reasonable to expect that $\frac{1}{r}\frac{dr}{dt}$ and $\frac{1}{M}\frac{dM}{dt}$ would be of the same order of magnitude; thus if the mass has not changed appreciably, the same would apply to the radius. Professor Eddington, in a private communication to me, suggests another argument. It is a fact of observation that for stars of small mass, such as the sun, the spectral type is closely correlated with luminosity and therefore with mass; hence if the mass has not changed appreciably the temperature and the luminosity must both have been practically as at present. Thus the radius and density cannot have changed much.

Let us, then, consider the consequences of the two extreme possibilities indicated, according to which the density of the primitive sun had respectively 1 and 0·047 times its present value, or, say, 1·3 and 0·06 gm./cm.^{3}. On both hypotheses the density was very much greater than the value adopted as a working hypothesis in my book, and

* Cf. *loc. cit.*, 329, equations (9) and (10).

† I am indebted for this point to an unknown reviewer, in *Nature*, **114**, 742–4, 1924.

some changes in the mode of development of the planets naturally follow. In the first place, whereas on my former theory the solar constant at the time of disruption was thirteen times as great as it is now, on the present theory no great change is to be contemplated, and what change has occurred is more likely to have been a rise than a fall. The high value taken for the solar constant implied that the sun must have cooled through the giant stage and the early parts of the dwarf stage before the surface of the earth became cool enough for an ocean to exist on it. Thus some doubt remained as to whether the time that passed before the formation of the ocean was a large or a small fraction of the whole age of the earth. On the present theory the development of an ocean began when the earth was only a few thousand years old.

Another change evidently required is in the masses of the nuclei that could have been formed by gradual condensation. I show in 2·4 (9) of my book that the minimum mass of a planet formed by gradual condensation from a gaseous filament is proportional to $c^3\rho^{-\frac{1}{2}}$, where ρ is the density of the ejected matter and c is the velocity of sound in this matter. It was estimated at 8×10^{27} grams, rather more than the mass of the earth. With the greatest distension now considered possible, c remains the same, but ρ is 1000 times what I previously obtained. Thus the minimum mass is now $2{\cdot}5\times10^{26}$ grams, a little more than that of Mercury. If we proceed to the other extreme, given by the present temperature and density of the sun, we have

$$c \propto T_e^{\frac{1}{2}} \propto r^{-1\frac{5}{7}} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\rho \propto r^{-3} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whence

$$m \propto c^3\rho^{-\frac{1}{2}} \propto r^{1\frac{9}{14}} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\propto T_e^{-0{\cdot}9} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

making the minimum mass now $1{\cdot}4\times10^{26}$ grams, distinctly less than that of Mercury.

I showed in 2·51 of my book that smaller masses, too light to retain all their material by gravitation, might liquefy in two different ways. If the density was above about 10^{-5}gm./cm.^{3}, they would expand adiabatically until either liquefaction was complete or they had reached this critical density. If the density was below this value there would be little expansion, and liquefaction would take place by radiation from the surface. The estimate of the critical density is independent of the condition of the primitive sun. The density of the ejected matter, however, is on the present hypothesis at least 0·06 gm./cm.^{3}. Thus adiabatic expansion would play the most prominent part in the cooling of the smaller masses. The density would, indeed, exceed the critical density so much that nearly the whole of the matter would liquefy before the critical density was reached. Cooling by radiation ceases therefore to be important in the liquefaction of the satellites.

On the Progression of Stellar Velocity with Absolute Magnitude.

By Gorakh Prasad, D.Sc.

(*Communicated by the Astronomer Royal for Scotland.*)

It is well known that there is a correlation between the velocities and absolute magnitudes of stars, but the determination of the rate of change of velocity with absolute magnitude presents unusual difficulties. Eddington and Miss Douglas * have shown that the systematic error in this quantity produced by the accidental errors of the magnitudes is very great. Thus the value of the increase of the cross-component (*i.e.* at right angles to the direction towards the solar apex) of the linear tangential velocity per unit absolute magnitude for the K giants has been shown by them to be 3·46 or 7·54 km. per sec. according as the probable error of the absolute magnitudes is taken to be $0^{M}\cdot3$ or $0^{M}\cdot4$. The value $0^{M}\cdot5$ for the probable error is not unlikely. In fact, the values of the probable errors for various groups of the stars considered by Eddington and Miss Douglas, as given by Adams and his collaborators,† range from $0^{M}\cdot53$ to $0^{M}\cdot32$. From the material given by Eddington and Miss Douglas the value of the increase of linear cross-velocity per unit absolute magnitude may be calculated to be 22·5 km. per sec. We see how sensitive this quantity is to a change in the adopted value of the probable error of the absolute magnitudes.

If the stars are grouped according to any characteristic in the value of which the absolute magnitudes, or the parallaxes derived from them, do not enter, the mean absolute magnitude for the group will have a far smaller accidental error, and therefore the value of the increase of linear cross-velocity per unit absolute magnitude will be more reliable. The function $H \equiv m+5+5 \log \mu$, where μ is the total proper motion, is a suitable characteristic for grouping the stars, since there is a relation between H and mean absolute magnitude, and since in the calculation of H neither the absolute magnitude nor the parallax is used.

The investigation of the relation between mean absolute magnitude and mean linear cross-velocity when the stars are grouped according to their value of H is the main object of this paper. The relation between mean absolute magnitude and mean total tangential velocity when the stars are grouped according to H can be immediately derived from the relation between H and mean absolute magnitude found in a previous paper of mine.‡ This will be taken up first.

1. *The Rate of Increase of Total Tangential Linear Velocity, with Absolute Magnitude deduced from the relation between H and M.*

Let the stars be grouped according to the value of their H, the range of the values of H within any one group being small. Consider any one group, and denote the mean of the values of H by H and the mean of the absolute magnitudes by M. Let the true (*i.e.* unaffected

* *Monthly Notices*, **83** (1923), 112.
† *Ap. J.*, **53** (1921), 27.
‡ *Monthly Notices*, **85** (1924 December).

by accidental errors) absolute magnitudes of the individual stars be denoted by M′. Put

$$\overline{M}=M'+x \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We suppose that x has a normal frequency distribution, the standard deviation being σ. Let the relation between H and $\overline{M}$ be

$$\overline{M}=a+bH+cH^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since c is small, this gives approximately

$$H=5A+(5B+1)\overline{M}+5C\overline{M}^2 \quad . \quad . \quad . \quad . \quad (3)$$

where

$$A=-\tfrac{1}{5}\left(\frac{a}{b}+\frac{a^2c}{b^3}\right),$$

$$B=-\tfrac{1}{5}\left(1-\frac{1}{b}-\frac{2ac}{b^3}\right),$$

and

$$C=-\tfrac{1}{5}\frac{c}{b^3}.$$

Bearing in mind that

$$H=m+5+5\log\mu$$

and

$$M'=m+5+5\log\pi,$$

we get from (3) and (1)

$$V=10^{A+B\overline{M}+C\overline{M}^2}\,.\,10^{0\cdot2x} \quad . \quad . \quad . \quad . \quad (4)$$

where $V=\mu/\pi$ and is the total linear tangential velocity, its unit being one astronomical unit per annum, or 4·74 km. per sec. Expanding $10^{0\cdot2x}$ in powers of x and taking the mean, we find that the mean velocity

$$\overline{V}=10^{A+B\overline{M}+C\overline{M}^2}(1+0\cdot106\sigma^2) \quad . \quad . \quad . \quad . \quad (5)$$

powers of σ higher than the third being neglected. We have therefore

$$\log\overline{V}=A+B\overline{M}+C\overline{M}^2+\log(1+\cdot106\sigma^2) \quad . \quad . \quad . \quad (6)$$

In cases in which B and C are small, we can express $\overline{V}$ in the form

$$\overline{V}=a'+b'\overline{M}+c'\overline{M}^2 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

by expanding $10^{B\overline{M}+C\overline{M}^2}$ in powers of $\overline{M}-M_1$, where M_1 is some convenient number nearly equal to the mean absolute magnitude of all the stars.

The case in which the relation between H and M is a straight line can be obtained from the above by putting $c=C=0$. The values of A, B, C for various spectral types as calculated from the values of a, b, c given in Table I. of my previous paper are shown below, V being supposed to be measured in km. per sec. The term $\log(1+\cdot106\sigma^2)$ is generally very small.

TABLE I.

Values of A, B, C *in formula* (6) *for various Spectral Classes,* V *being measured in km. per sec.*

Spectral Class.	Range of the Values of M.	A.	B.	C.
B8, B9, Ao, A2, A3, A5	0·2 to 2·5	+0·89	+0·291	
Fo, F2, F5, F8	0·2 ,, 5·1	+0·93	+0·081	+·0101
Fo, F2, F5, F8	2·9 ,, 5·1	+0·18	+0·360	
Go, G5	−1·0 ,, 6·2	+1·17	+0·042	+·0057
Go, G5	3·7 ,, 6·2	+0·63	+0·226	
Ko, K2 Giants	−0·1 ,, 1·6	+0·91	+0·451	
Ko, K2, K5 Giants	−0·3 ,, 1·8	+1·00	+0·349	
Ko, K2 Dwarfs	3·4 ,, 8·4	+1·50	+0·031	
Ko, K2 Dwarfs	1·6 ,, 8·4	+1·57	+0·021	
K5, M*a*, M*b*, M*c* Giants	−2·1 ,, 2·1	+1·25	+0·109	
M*a*, M*b*, Mc Giants	−1·3 ,, 1·1	+1·33	+0·260	
K5, M*a*, M*b*, Mc Dwarfs	7·8 ,, 10·5	+0·49	+0·133	

2. *The Increase of Linear Cross-Velocity with Absolute Magnitude.*

The above gives an increase of about 21 km. per sec. per unit absolute magnitude for the K giants. This is very high. In order to get results which could be compared with those of Eddington and Miss Douglas, the increase of linear cross-velocity with absolute magnitude was calculated for the stars considered by these authors, viz. the stars from type G8 to K2 inclusive (Mount Wilson estimated spectrum) brighter than absolute magnitude $3^{M}\cdot5$ contained in the *Astrophysical Journal*, **53** (1921), 13, excluding such stars as are not included in Boss's *Preliminary General Catalogue.*

The proper motions were taken from Boss. For calculating the cross-component of the proper motion, the position-angle of the solar ant-apex was read off from the charts published by Smart,* in which the position of the ant-apex is taken to be R.A. $6^{h}\ 0^{m}$, Dec. $-34°$. The linear cross-velocity v was obtained by dividing the cross-component

TABLE II.

Mean Values of H, M, *and v.*

Number.	$\overline{H}$.	$\overline{M}$.	$\bar{v}$ (all).	$\bar{v}$ (−3).	$\bar{v}$ (−6).
1	−0·09	0·46	0·73	0·45	0·39
2	2·42	0·96	1·50	1·16	1·05
3	4·07	1·19	2·62	2·39	2·20
4	5·23	1·07	4·14	3·82	3·57
5	7·00	1·64	5·69	5·00	4·63

* "Charts giving the Angular Distances of Stars from, and the Position Angles relative to, the Ant-apex of the Solar Motion," by W. M. Smart. London, Royal Astronomical Society, 1923. See also *Monthly Notices*, **83** (1923), 465.

of the proper motion by the spectroscopic parallax. The stars, 285 in number, were arranged in order of H and divided into five groups, each containing the same number of stars. The mean values of H, M, and v are shown in Table II., the unit of v being a velocity of 4·74 km. per sec. In order to reduce the accidental effect of a few excessive motions, the table also contains the mean values of v when (1) the three largest velocities in each group, and (2) the six largest velocities in each group are omitted. Correcting for the effect of the omission of the largest velocities in exactly the same way as has been done by Eddington and Miss Douglas, the formulæ giving $\bar{v}$ in terms of $\overline{M}$ are

$$\bar{v}=-7{\cdot}1+19{\cdot}8\overline{M} \text{ km. per sec. (all stars) . . . (8)}$$
$$\bar{v}=-8{\cdot}2+20{\cdot}4\overline{M} \text{ ,, ,, (15 rejected) . . (9)}$$
$$\bar{v}=-8{\cdot}4+20{\cdot}8\overline{M} \text{ ,, ,, (30 rejected) . . (10)}$$

the unit of v being now one km. per sec. The three formulæ agree very closely. Taking the middle one as definitive, we find an increase of 20·4 km. per sec. per unit absolute magnitude. This may be compared with the values 3·5, 7·5, and 22·5 km. per sec. of Eddington and Miss Douglas mentioned before, and the values 3·2 km. per sec. and 1·4 km. per sec. of Adams, Strömberg, and Joy * for the increase per unit absolute magnitude of the total space velocity and radial velocity respectively. It should be mentioned that in the latter investigation the effect of the accidental errors of the absolute magnitudes is not allowed for.

The probable error of the coefficient of $\overline{M}$ in equation (9) is $\pm 4{\cdot}2$ km. per sec., which is 21 per cent. of the quantity sought. Thus the present determination is of the same order of accuracy as that of Eddington and Miss Douglas; the probable error in their case is 19 per cent. when it is assumed that the value of the probable error of the absolute magnitudes is accurately known to be $0^{M}{\cdot}3$.

The points for the five groups and the straight line given by equation (9) are shown in fig. 1.

There are two factors tending towards a high value of the rate of change of linear cross-velocity with absolute magnitude. Firstly, when the groups are formed according to H and the mean of the absolute magnitudes is taken, instead of the groups being formed according to the absolute magnitude itself, the apparent dispersion of the absolute magnitudes caused by the accidental errors is almost eliminated. The total range of the absolute magnitudes being thus diminished, the rate of increase of velocity with absolute magnitude comes out high. Secondly, there is not perfect correlation between linear cross-velocity and absolute magnitude, and, as is well known,† in cases of imperfect correlation the straight lines obtained according as the groups are formed with respect to one or the other of the two characteristics between which a relation has to be found have different slopes, and

* *Ap. J.*, **54** (1921), 9.

† See, for example, Brunt, *The Combination of Observations*, Cambridge, 1917, p. 155.

the weaker the correlation between the two characteristics, the greater is the difference in their slopes. Apparently, grouping the stars according to H has an effect similar to grouping them with respect to v. An attempt to find the increase of velocity with absolute magnitude by grouping the stars with respect to v seems hopeless, because v is much more uncertain than H and because the use of the parallax derived from the absolute magnitude in calculating it further complicates the problem. The masking of the progression of velocity with absolute magnitude on account of the accidental errors in M and v is well shown

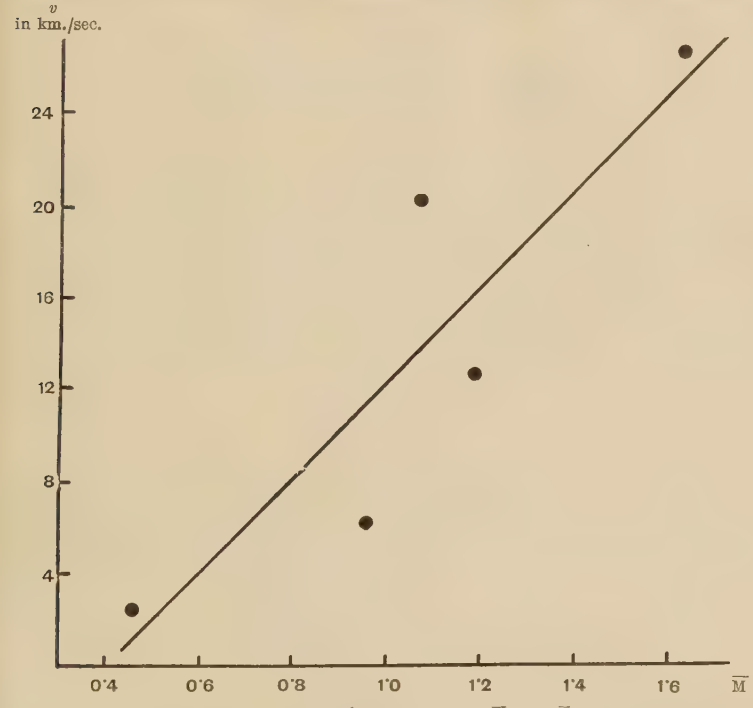


FIG. 1.—Relation between $\bar{M}$ and $\bar{v}$.

by the "scatter diagram" (fig. 2), in which the points corresponding to individual stars are plotted. Three points with values of v greater than 60 km. per sec. lie outside the figure.

We see that *when the stars are grouped according to H, the increase of the cross-component of the linear tangential velocity has a much higher value than when they are grouped according to the observed absolute magnitude. Making an allowance for the effect of the accidental errors of the absolute magnitudes brings the two results into closer agreement. For complete agreement, the value of the probable error of the absolute magnitudes should be a little less than* $0^{M}\cdot5$, a value for or against which there is little evidence.

On account of the very imperfect correlation between linear cross-

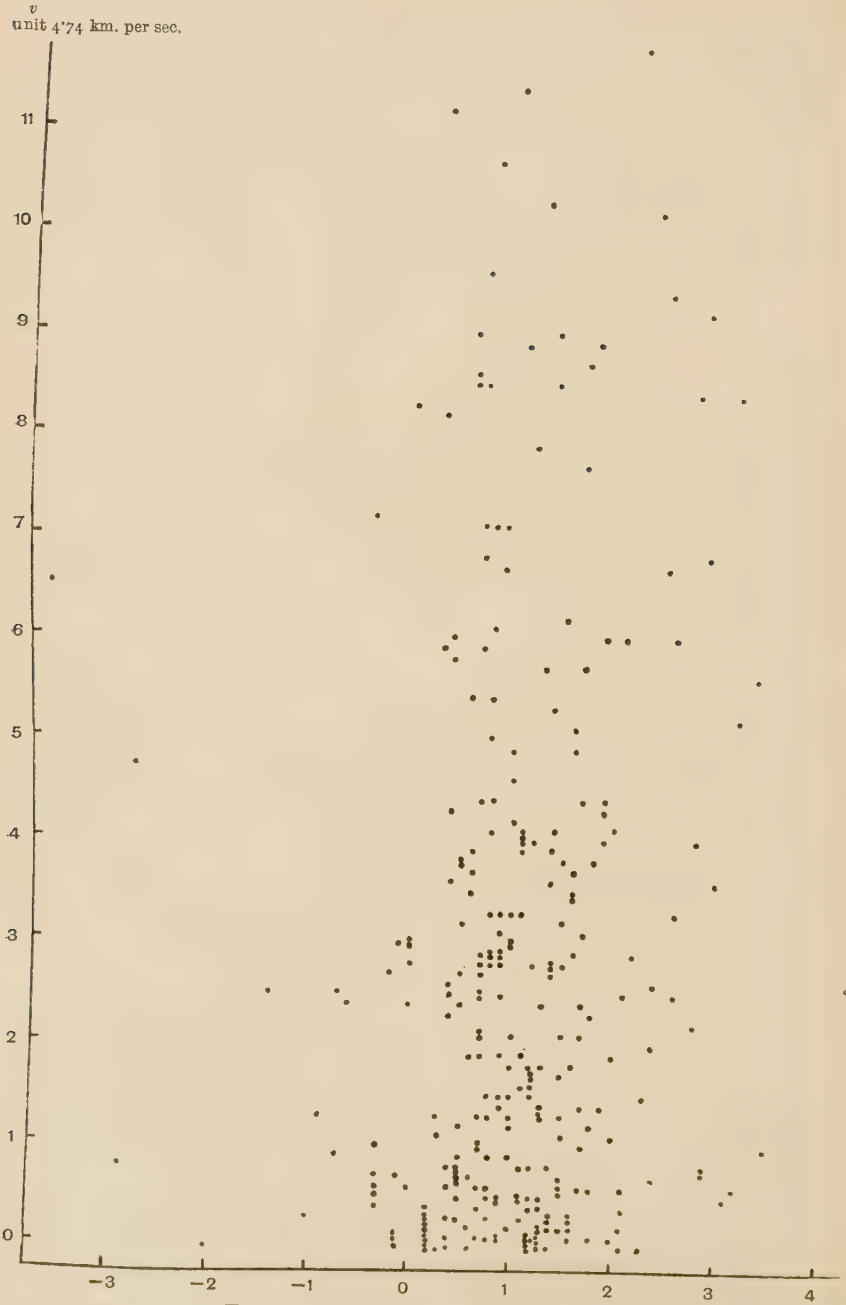


FIG. 2.—"Scatter Diagram" of M and *v*.

velocity and absolute magnitude, little reliance should be placed on the mean linear cross-velocity for a small group of stars calculated from the numerical formulæ here derived.

In conclusion, I wish to express my gratitude to Professor Sampson for his kind interest in the preparation of this paper.

Royal Observatory, Edinburgh:
1925 *January* 19.

The Cosmogonic Time-Scale and Binary Stars.

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

§ 1. In his remarkable paper presented to the Society in 1924 November (*M.N.*, **85**, 2), Dr. Jeans discusses several cosmogonic problems on the basis of a time-scale of 7×10^{12} years. The time-scale itself is derived by developing the idea of the secular decrease of stellar mass, the radiation of energy being associated with loss of mass. Professor Eddington's paper "On the Relation between the Masses and Luminosities of the Stars" (*M.N.*, **84**, 308) supplies an approximate formula

$$\frac{dM}{dt}=-\alpha M^3 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

for stars of mass (M) between 4 and 1, in terms of the solar mass. In this equation the constant α is evaluated from known quantities associated with the Sun. The integration of (1) leads to the time-scale; in particular, if it is supposed that the Sun had, at one time, a mass 4, the interval between then and now is found, from (1), to be $7{\cdot}1\times10^{12}$ years.

We write, more generally,

$$\frac{dM}{dt}=-\kappa L \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where L is the luminosity of the star and κ a constant to be evaluated later in terms of the particular units employed.

Now the principal equations of Professor Eddington's theory are

$$L=\text{const.}\times M^{\frac{1}{2}}(1-\beta)^{\frac{3}{2}}\mu^{\frac{1}{2}}T_e^{\frac{4}{5}} \quad . \quad . \quad . \quad . \quad (3)$$

in which β is given in terms of the mass M by

$$1-\beta=JM^2\beta^4 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$J=0{\cdot}00309\mu^4$$

and μ is the average molecular weight.

The equation (3) contains T_e—the effective stellar temperature—and consequently there is introduced a varying correction on this account.

For the range of temperatures 5000° to 12000° the factor introduced by T_e varies from 1 to about 2, and a time-scale derived by putting T_e= constant will consequently be overestimated. But since the star races through the earlier part of its career, the major part of the time-scale is due to the change in temperature from, say, 6000° or 7000° (if we are considering the Sun) to 5000°. If equation (3) is written

$$L=AM^{\frac{1}{2}}(1-\beta)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where A is regarded as constant, the time-scale derived will not be greatly overestimated.

Professor Eddington has also given another formula (p. 328) for the luminosity L; it is

$$L=\frac{B(1-\beta)^2}{\beta} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and from this equation he calculates the relative time spent between successive limits of absolute magnitude or, alternatively, of mass.

The equations (1), (2), (5), and (6) give us, then, three methods of estimating the time-scale. The equation (1) has the merit of simplicity, and it is of interest to see how it compares with the formulæ (5) and (6). Moreover, in the second part of this paper the equation (2) is applied in considering the secular change in the dimensions of binary star orbits, and it is clearly of advantage to ascertain the relative performances of the three methods before embarking upon this subject.

§ 2. Taking the mass of the Sun as unity and the year as the unit of time, we find from (1) that $\alpha=6{\cdot}5\times10^{-14}$ and, consequently,

$$t_2-t_1=7{\cdot}7\times10^{12}\left(\frac{1}{M_2^{\,2}}-\frac{1}{M_1^{\,2}}\right),$$

where M_1, M_2 are the masses of the Sun at times t_1, t_2. The time-scale ($M_\odot=4$ to $M_\odot=1$) is, of course, that found by Dr. Jeans, viz. $7{\cdot}1\times10^{12}$ years.

§ 3. Considering now equation (6) and putting, for the Sun as at present,

$$M=1, \quad L=1, \quad 1-\beta=0{\cdot}0497,$$

we evaluate directly the constant B and obtain

$$\frac{dM}{dt}=-380\kappa M\frac{(1-\beta)^2}{\beta} \quad . \quad . \quad . \quad . \quad (7)$$

where κ has the same value as α in § 2, viz. $6{\cdot}5\times10^{-14}$. Eliminating M between (4) and (7) and integrating, we have

$$t_2-t_1=2{\cdot}02\times10^{10}\left[\frac{3\epsilon_2+\frac{1}{2}}{\epsilon_2^{\,2}}-\frac{3\epsilon_1+\frac{1}{2}}{\epsilon_1^{\,2}}\right] \quad . \quad . \quad . \quad (8)$$

in which we have written

$$\epsilon=1-\beta \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Equation (8) gives the second method of evaluating the time-scale.

§ 4. Coming now to the third method, we have from (2) and (5)

$$\frac{dM}{dt}=-\kappa A M^{\frac{1}{6}}(1-\beta)^{\frac{3}{2}},$$

and eliminating M between this equation and (4), we obtain

$$\frac{d\beta}{dt}=\frac{2\kappa A}{J^{\frac{1}{6}}}\frac{\beta^{\frac{1}{6}}(1-\beta)^{\frac{23}{10}}}{4-3\beta}.$$

In terms of the units used already, the constant J of equation (4) is found to be 0·06092 and the constant A of (5) to be 89·45. κ has the same value as in § 3. Inserting these values and putting $\epsilon=1-\beta$, we have

$$t_2-t_1=4\cdot92\times10^{10}\int_{\epsilon_2}^{\epsilon_1}\frac{1+3\epsilon}{(1-\epsilon)^{\frac{1}{6}}\epsilon^{\frac{23}{10}}}d\epsilon \quad . \quad . \quad . \quad (10)$$

In the physical problem ϵ is positive and less than 1. If, for example, we limit the range of masses from 11 to $\frac{1}{4}$, the range of ϵ is from 0·50 to 0·004. The function under the integral sign can then be expanded in series and integrated term by term, the necessary conditions of convergence being amply satisfied. The result is

$$t_2-t_1=1\cdot97\times10^{12}\left[\epsilon^{-\frac{13}{10}}\left\{\frac{1}{68}+\frac{4\epsilon}{35}-\sum_{r=2}(A_r\epsilon^r)\right\}\right]_{\epsilon_1}^{\epsilon_2} \quad . \quad . \quad (11)$$

where

$$A_r=\frac{5r-1}{(10r-17)r!\,5^r}\cdot\prod_{s=1}^{r-1}(5s-4).$$

For the largest value of ϵ considered, viz. 0·45, only the first four terms of the summation were required.

For example, if at t_1

$$M_1=4\cdot137 \text{ and } \epsilon_1=0\cdot28$$

and at t_2

$$M_2=1\cdot0 \text{ and } \epsilon_2=0\cdot05,$$

the equation (11) gives the result

$$t_2-t_1=9\cdot0\times10^{12} \text{ years.}$$

§ 5. If we now suppose that the Sun has had, in the course of its evolutionary career, masses M_1 given in the following table (corresponding to a sequence t_1 in time), and if t_2 corresponds to its present mass $M_2=1$, we have the values (t_2-t_1) in years as given by the three procedures:

	M_1		
	8·984.	4·137.	1·582.
Paragraph 2 . .	$7\cdot6\times10^{12}$	$7\cdot2\times10^{12}$	$4\cdot6\times10^{12}$
,, 3 . .	5·1	4·9	3·7
,, 4 . .	9·1	9·0	5·9

The values of ϵ corresponding to the above sequence of masses M_1 are 0·45, 0·28, and 0·10 respectively.

It is noticed that the time-scale derived in § 4 is from 20 to 30 per cent. longer than that obtained by Dr. Jeans. If we consider a star whose present mass is 0·512 (ϵ=0·015) and absolute magnitude 7·8, and further suppose that it has evolved through the sequence of masses 4·137, 1·582, and 1·0, we find the corresponding time-intervals to be

$$6{\cdot}2\times10^{13},\quad 5{\cdot}9\times10^{13},\quad \text{and}\quad 5{\cdot}3\times10^{13}\ \text{years}.$$

If we invert the calculation and assume that the Sun has existed for 9×10^{12} years, during which its mass has decreased from 4 to 1, then the change of mass of a star (present mass 0·51) has been almost insignificant; actually, the diminution of mass is of the order 0·05; that is to say, the mass has decreased from 0·56 to 0·51.

Making the calculations now for a star whose present mass is 0·2583 (ϵ=0·004) and absolute magnitude 10·8, we find that if it has evolved through the sequence of masses 1·582, 1·0, and 0·5117, then the corresponding time-intervals are

$$5{\cdot}6\times10^{14},\quad 5{\cdot}5\times10^{14},\quad \text{and}\quad 5{\cdot}0\times10^{14}\ \text{years}.$$

These figures are, to a certain extent, underestimated, for during the greater part of the star's career the effective temperature is in the region 3000° to 4000°, and the temperature factor in equation (3) is less than that used implicitly in the general equations.

Passing now to larger stellar masses, the calculations show that the interval required for a star to run through its course—for example, from mass 10 to mass 4—is 4×10^{11} years.

§ 6. Evidently the theory of secular decrease in mass is unable to solve the problem of the origin of the stellar system as at present constituted. There seem to be two aspects of the problem :—

(1) A more or less simultaneous origin of the majority of stars, with a time-scale of the order of 10^9 or 10^{10} years, during which the most massive stars at the beginning have had sufficient time to decrease notably in mass, say to 4 or 2; the Sun has decreased from about 1·2 to 1·0, and the M dwarfs have changed in mass hardly at all. On this view the M giants and other stars of very great luminosity must be regarded as intruders, formed comparatively recently outside the galactic system and gaining admission owing to circumstances connected with their origin. It is difficult, however, to see how this can be reconciled with the well-known spacial-velocity characteristics of the stars in general.

A further difficulty is the origin of dwarf stars of mass, say, about $\frac{1}{2}$, at present. With the time-scale contemplated in this section, the original mass of a dwarf star would be practically the present-day mass. The observational evidence seems to be, on the whole, that at the bottom end of the giant sequence the masses apparently come to a lower limit in the neighbourhood of 2 or 3. The time-scale would thus be insufficient so far as it concerns the feebler dwarfs. Thus the supposition of a common origin—with the time-scale of 10^9 or 10^{10} years—must exclude the dwarf from the foundational system. Accordingly, they must be regarded—on this hypothesis—as intruders of

venerable age formed outside the bounds of the stellar system at a very much more remote time.

(2) On the other hand, we may regard the dwarfs as the original members of the stellar system with a time-scale of the order of 5×10^{14} years. The Sun must then be supposed to have joined comparatively late in cosmogonic history. The giants, formed, still later, in external space, would, on this supposition, be comparatively recent acquisitions. On this view, the stellar system is a growing congeries of stars, acquiring continually new members from outside. The approaching velocities of the globular clusters, the high-velocity groups such as the Me variables, the open clusters, all seem to offer some evidence on this point. The idea of continuous growth precludes the possibility of the stellar system having reached, even at the present time, a steady state. The expansion of the stellar universe would still continue, whatever aspect we favour; or, stated otherwise, the dimensions of the stellar system would be much less, 10^{12} years ago, than at present, but the probability of encounters in bygone cosmogonic ages would not be so great as previously estimated, owing to the very much smaller number of stellar inhabitants.

§ 7. A comparison of the different methods of deriving the time-scale shows that Dr. Jeans' formula—(1) of this paper—is, with the restrictions mentioned, a good approximation, and in more complicated problems it has the additional merit of simplicity. Accordingly, I have adopted it in discussing binary star orbits.

Let m, m_1 be the masses of the components of a binary star. We shall assume that m is not greater than m_1. The motion of m about the common centre of mass G is the same as the motion of a particle about a fixed mass M at G where M is given by

$$\mathrm{M}=\frac{m_1{}^3}{(m+m_1)^2} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Similarly the motion of m_1 is determined with reference to the mass-function M_1 given by

$$\mathrm{M}_1=\frac{m^3}{(m+m_1)^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Dr. Jeans has obtained the general integrals of motion, viz.

$$\mathrm{M}a=\mathrm{C} \quad \text{and} \quad \mathrm{M}_1a_1=\mathrm{C}_1 \quad . \quad . \quad . \quad . \quad (14)$$

where a, a_1 are the semi-major axes of the elliptic motion about the common centre of mass and where C, C_1 are constants.

It is to be remembered that M has now a different signification from what it had in the first part of this paper.

Equation (14) shows that if M_1 is decreasing, a_1 is consequently increasing. But it may be noticed that M_1, for example, does not necessarily decrease with the time. Actually, we have from (13)

$$\frac{d\mathrm{M}_1}{dt}=\frac{m^2}{(m+m_1)^2}\cdot\frac{dm}{dt}\left[(3m_1+m)-2m\frac{dm_1}{dt}\Big/\frac{dm}{dt}\right] \quad . \quad . \quad (15)$$

$\frac{dm}{dt}$ is, on the theory of secular decrease of mass, of course negative and $\frac{d\mathrm{M}_1}{dt}$ is positive if

$$2m\frac{dm_1}{dt}\Big/\frac{dm}{dt} > (3m_1+m).$$

Using (1) and putting $u=m/m_1$ (the mass-ratio), we have the above inequality in the form

$$u^3+3u^2<2.$$

The equation $u^3+3u^2-2=0$ has one positive root at 0·73 (the remaining roots are negative). Hence we conclude that $\frac{d\mathrm{M}_1}{dt}$ is positive if $u=\frac{m}{m_1}<0{\cdot}73$.

It is easily shown that when $\frac{d\mathrm{M}_1}{dt}$ is positive or zero, $\frac{d\mathrm{M}}{dt}$ is essentially negative, and that when the former is negative then the latter is, likewise, always negative.

Thus in considering the orbits of the two components about the centre of mass, the dimensions of the orbit of the more massive star decrease with the time if the mass-ratio is less than 0·73, whereas the dimensions of the orbit of the less massive component increase whatever the value of u. It will be shown later—as can readily be seen on more general grounds—that the separation of the components continually increases whatever the value of the mass-ratio may be. The mass-ratio is evidently, however, of great importance in connection with the rate of expansion of a binary system.

§ 8. For simplicity I shall consider circular orbits only. Let A denote the separation of the components at any time so that

$$\mathrm{A}=a+a_1.$$

From (14) we have

$$\frac{d\mathrm{A}}{dt}=-\left[\frac{a}{\mathrm{M}}\frac{d\mathrm{M}}{dt}+\frac{a_1}{\mathrm{M}_1}\frac{d\mathrm{M}_1}{dt}\right].$$

We have also

$$ma=m_1a_1 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Introducing the mass-ratio u and using (14), we obtain after some reduction

$$\frac{d\mathrm{A}}{dt}=a\mathrm{C}m_1(1+u)^2(1+u^3) \quad . \quad . \quad . \quad . \quad (17)$$

which shows that the separation of the components increases continually with the time.

Let now n, n_1 denote the masses of the components at time $t=0$. Then from (1) we have

$$2at=\frac{1}{m^2}-\frac{1}{n^2} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and a similar equation in m_1, n_1.

From (18) and its fellow we have

$$u^2 \equiv \frac{m^2}{m_1^2} = \frac{u_0^2(1+2\alpha n_1^2 t)}{1+2\alpha n^2 t} \quad . \quad . \quad . \quad . \quad (19)$$

where $u_0 \equiv \frac{n}{n_1}$ is the ratio of the masses at time $t=0$.

From (19) t can be expressed as a function of u, and (17) is then easily transformed into

$$\frac{dA}{du} = \frac{C(1-u_0^2)^{\frac{1}{2}}}{n} \cdot \frac{(1+u)^2(1+u^3)}{(1-u^2)^{\frac{3}{2}}} \quad . \quad . \quad . \quad (20)$$

in which the coefficient α has disappeared explicitly.

Actually, from (19) we have

$$\frac{du}{dt} = \frac{\alpha n^2 (1-u^2)^2}{u(1-u_0^2)},$$

which shows that the mass-ratio increases with the time. This result can be derived qualitatively from equation (1), for the more massive component radiates its mass away proportionately more than does the less massive component, so that the mass-ratio increases continuously with the time. This will be referred to later.

We now express the constant C in terms of the separation A_0 of the components at time $t=0$. If b, b_1 denote the values of a, a_1 at $t=0$, we have

$$A_0 = b + b_1.$$

Also, $nb = n_1 b_1$, so that

$$A_0 = b(1+u_0).$$

Now

$$C = ma = nb = \frac{n_1^3 b}{(n+n_1)^2} = \frac{n_1 b}{(1+u_0)^2}.$$

Hence, inserting the value of b, viz. $A_0/(1+u_0)$, in the last expression, we have

$$C = \frac{A_0 n_1}{(1+u_0)^3}.$$

Hence from (20)

$$\frac{dA}{du} = A_0 \frac{(1-u_0^2)^{\frac{1}{2}}}{u_0(1+u_0)^3} \cdot \frac{(1+u)^2(1+u^3)}{(1-u^2)^{\frac{3}{2}}} \quad . \quad . \quad . \quad (21)$$

Put $u = \sin\theta$ in (21) and the integration is easily effected. The result is

$$\frac{A}{A_0} = 1 + G(u_0)[F(u) - F(u_0)] \quad . \quad . \quad . \quad (22)$$

where

$$G(u_0) = \frac{(1-u_0^2)^{\frac{1}{2}}}{u_0(1+u_0)^3}$$

and

$$F(u) \equiv F(\theta)$$

$$= 4(1+\sin\theta)\sec\theta + \cos\theta(3+\sin\theta) - 4\theta - \tfrac{1}{3}\cos^3\theta.$$

In the following table the second and third columns give the values of $F(u)$ and $G(u_0)$ calculated for the mass-ratios 0·1 to 0·95. The remainder of the table gives the values of $\frac{A}{A_0}$ for the series of initial values 0·1 to 0·8 of u_0 (horizontally) and for the final values of u (vertically) from 0·1 to 0·95. Thus for a binary star whose components initially have a mass-ratio $u_0=0·2$, the ratio of the final separation of the components (when the mass-ratio has increased to the value given by $u=0·7$) to the initial separation is 6·9. If the mass-ratio is finally given by $u=0·9$, the initial ratio being the same, viz. 0·2, the dimensions of the system have been extended, during the process, in the ratio 23·4 to 1.

The maximum extension of a binary system shown in this table is in the ratio 108·6 to 1, corresponding to the values 0·1 and 0·95 of u_0 and u respectively.

TABLE I.

u.	$F(u)$.	$G(u_0)$.	0·1.	0·2.	0·3.	0·4.	0·5.	0·6.	0·7.	0·8.
0·1	6·78	7·47								
0·2	6·91	2·90	2·0							
0·3	7·09	1·45	3·3	1·5						
0·4	7·32	0·84	5·1	2·2	1·3					
0·5	7·65	0·51	7·5	3·1	1·8	1·3				
0·6	8·14	0·33	11·2	4·5	2·5	1·7	1·3			
0·7	8·94	0·21	17·1	6·9	3·7	2·4	1·7	1·3		
0·8	11·11	0·13	33·3	13·2	6·8	4·2	2·8	2·0	1·5	
0·9	14·63	0·07	59·6	23·4	11·9	7·1	4·6	3·1	2·2	1·2
0·95	21·18	0·04	108·6	42·4	21·4	12·5	7·9	5·2	3·5	1·7

It should be remarked again that this table is based on equation (1); consequently, the table is valid only for initial and final values of the masses which are adequately represented by (1). For masses outside the limits, *i.e.* for very large or very small masses—as compared with the Sun—the table may serve as a standard of reference, giving an approximation, at least, to the changes occurring during a cosmogonic cycle.

§ 9. The interval of time during which the mass-ratio has increased from u_0 to u, or the dimensions of the system have grown according to the values given in Table I., is readily found from the two equations (18). For the sake of convenience in dealing with the great time-intervals which the theory of the secular decrease in mass involves, we put in (18) $2a=1$, so that the unit of time is $7·7\times10^{12}$ years—the interval required for a star to pass from an infinitely large mass to the present-day mass of the Sun. This unit may be referred to appropriately as the "Cosmogonic Unit" of time. Equation (18) can then be written

$$T=\frac{1}{m^2}-\frac{1}{n^2} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

in which T is the time expressed in cosmogonic units. There is a similar equation in m_1, n_1. These equations lead to (19), in which we now suppose that $2a=1$ and that T is written for t. Expressing T in terms of u and u_0, we have

$$n^2\mathrm{T}=\frac{u^2-u_0^2}{1-u^2}.$$

The following table gives the values of the function (n^2T) for the previous range in the values of u_0 and u, or, when considered alongside Table I., it gives (if n is known) the time required for any extension of the dimensions of a binary system.

TABLE II.

The Values of (n^2T).

$u_0\rightarrow$ $u\downarrow$	0·1.	0·2.	0·3.	0·4.	0·5.	0·6.	0·7.	0·8.	0·9.
0·2	0·03								
0·3	0·09	0·05							
0·4	0·18	0·14	0·08						
0·5	0·32	0·28	0·21	0·12					
0·6	0·55	0·50	0·42	0·31	0·17				
0·7	0·94	0·88	0·78	0·65	0·47	0·25			
0·8	1·75	1·67	1·53	1·33	1·08	0·78	0·42		
0·9	4·21	4·05	3·79	3·42	2·94	2·37	1·68	0·89	
0·95	9·15	8·84	8·33	7·61	6·70	5·56	4·23	2·69	0·95

If, for example, we assume that the initial mass n of the less massive component is 3, the whole evolutionary series contemplated in Table I. is shown by Table II. to take place within one cosmogonic unit of time. As regards the more massive component of such a system, if $u_0=0{\cdot}1$, then $n_1=30$, and consequently the equation (1) is probably very far from being a true expression for the secular decrease in mass of such a star. But m_1 decreases, initially, in any case, very rapidly, and consequently u increases rapidly at first, so that the estimates in Table II. can hardly be seriously in error.

§ 10. The observational evidence with regard to the increase of the mass-ratio with advancing age appears to be corroborative so far as it goes. Aitken's table of 31 spectroscopic binaries is, at least, satisfactory from this point of view—the mass-ratio is found to increase from 0·7 for type B binaries to 0·9 for types F and G (*The Binary Stars*, p. 207). Jeans finds confirmation—although not completely convincing, owing to the meagreness of the material—of the theory of secular decrease of mass (*Nature*, 1924 December 6). A similar conclusion is reached by H. Vogt (*Zeitschrift für Physik*, **26**, 2, 139, 1924 August) with respect to the mass-ratio of 93 visual double stars, and by G. Shajn (*M.N.*, **85**, 245). Shajn's argument is based on a direct application of Professor Eddington's equation (3) of this paper. The material considered by Shajn is more extensive, but it shows the same features as that of Vogt's paper.

Having regard to the nature and number of the observed binary stars for which the mass-ratios have been determined, it would seem that the results collected in Tables I. and II. are not inconsistent with observation, and that the theory of the secular decrease of stellar mass does afford an explanation—taken in conjunction with the increased time-scale—of the observed phenomena of binary stars so far as the mass-ratio, at least, is concerned.

§ 11. Can the theory account further for the evolution of visual binaries from spectroscopic binaries ? Jeans answers this question in the negative. With the data of Tables I. and II. we can submit the question to a further test. We consider only circular orbits, and disregard the problems associated with the progression of the magnitude of the eccentricity from the spectroscopic to the visual binaries.

Let us assume that a visual binary, period P years and separation A astronomical units, has evolved, through the cosmogonic ages, from a spectroscopic binary, period P_0 and separation A_0. Then with the notation of this paper for the masses we have

$$A_0^3=P_0^2(n+n_1)$$

and

$$A^3=P^2(m+m_1),$$

so that

$$\left(\frac{P}{P_0}\right)^2=\left(\frac{A}{A_0}\right)^3\left(\frac{n+n_1}{m+m_1}\right) \quad . \quad . \quad . \quad . \quad (24)$$

If we assume initial values for m and m_1 and consider further the changes after an interval of T cosmogonic units of time, we have in (24), combined with Tables I. and II., a means of determining the period of the " visual " binary.

Let us suppose, for example, that the original masses of the components of a spectroscopic binary are 30 and 3, so that $u_0=0{\cdot}1$. After an interval given by $T=0{\cdot}5$ cosmogonic units we have from (23), $m_1=1{\cdot}414$ and $m=1{\cdot}280$, so that $u=0{\cdot}88$. (The mass-ratio u can be found also by interpolation in Table II.) We have now, in equation (24), $m+m_1=2{\cdot}69$ and $n+n_1=33$. Consequently we are enabled by (24) to calculate the period of the binary after an interval 0·5 cosmogonic units of time if we are given the initial period, for the value of $\frac{A}{A_0}$ is found from Table I. to be about 50. Adopting 10 days as the period of the spectroscopic binary, we find that the period of the " visual " binary is 35 years.

If we take, now, the cosmogonic interval to be 0·9, we obtain the results

$$m=0{\cdot}995 : m_1=1{\cdot}054 : u=0{\cdot}94 : \frac{A}{A_0}=96.$$

And the period of the " visual " dwarf binary is now about 100 years.

If the masses still further decrease, say to 0·5, a rough calculation by (14) and (24) yields a period of 400 years.

§ 12. Spectroscopic binaries of mass much greater than that assumed

in § 11 are known. According to Dr. J. S. Plaskett, the minimum masses of the components of B.D. +6° 1309 are 75·6 and 63·3 (*M.N.*, **82**, 453). More recently, H. Ludendorff has suggested, on reasonable grounds, that the masses of the components of the spectroscopic binary υ Sagittarii are at least 260 and 54 respectively, so that the mass-ratio is 0·2 (*Sitz. der Preuss. Acad. der Wiss.*, **9**, 67, 1924 April). It is thus the most massive binary system known—if we leave the inclinations of the orbits out of consideration. The period is 138 days (*L.O.B.*, No. 267). Although equation (1) is not applicable to such a massive system, it is easily seen from (14) that the extension of the orbital dimensions must be extremely rapid and the increase in the period equally so.

The period and the sum of the masses of a visual binary define the linear dimensions of the orbit; whether the examples considered can be observed as "visual" binaries depends naturally on the parallax of the respective systems.

In conclusion, it may be remarked that the theory of secular decrease in mass has certain potentialities; given, originally, a sufficiently massive spectroscopic binary, the time-scale is amply sufficient to allow development into a visual binary, with a period of the usual magnitude.

Observatory, Cambridge:
1925 March 10.

The Systematic Motions of Faint Stars. (Fourth Paper.)

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

§ 1. My previous papers on this subject are to be found in *Monthly Notices*, **84**, 3, 123, 481. The methods described there have been followed in the present paper, and it will be sufficient to give the results obtained for the five regions considered here. The regions are as follows:—

	Region.	R.A. (1900)	Dec.	Interval (Years).	Number of Stars.	Limiting Magnitude (H.S.).
		h m	°			m
I.	Capella	5 9	+45·9	21·0	181	12·0
II.	β Pegasi	22 59	+27·5	19·3	50	11·5
III.	"Eros 90"	1 35	+52·8	23·0	61	11·0
IV.	"Eros 102"	1 32	+45·3	23·0	45	10·5
V.	"Eros 103"	1 33	+45·0	23·0	45	10·5

The third and fourth columns give the approximate positions of the plate-centres for 1900. The last column gives the approximate magnitudes (Harvard scale) of the faintest stars on the plates: these have been derived from Van Rhijn's Table IV. (*Groningen Publications*, **27**).

Many of the regions photographed by Mr. Hinks during the Eros

campaign of 1900–1 show a fair number of stars ; three of these regions (III., IV., V. above) have recently been re-photographed, and the

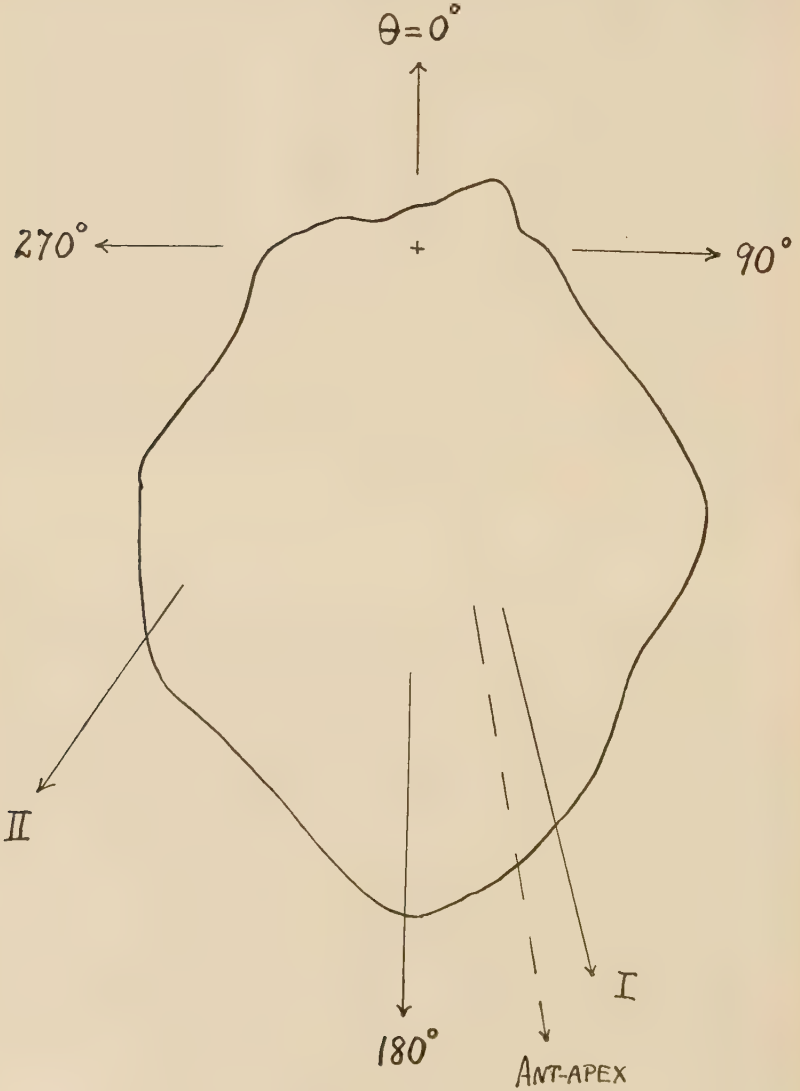


FIG. 1.—Capella.

results are given below. It should be possible to repeat all the Eros fields in the immediate future—certainly the results obtained from III.–V. are distinctly encouraging. It may be added that several proper motions of 10″ to 20″ per century have been detected in these

regions. The series of Eros plates is, in the main, an overlapping series, and this feature should be of great interest in the comparative study of proper motions derived from adjoining regions. Regions IV. and V. of this paper overlap to a large extent, and a brief discussion of the doubly observed proper motions will be found below.

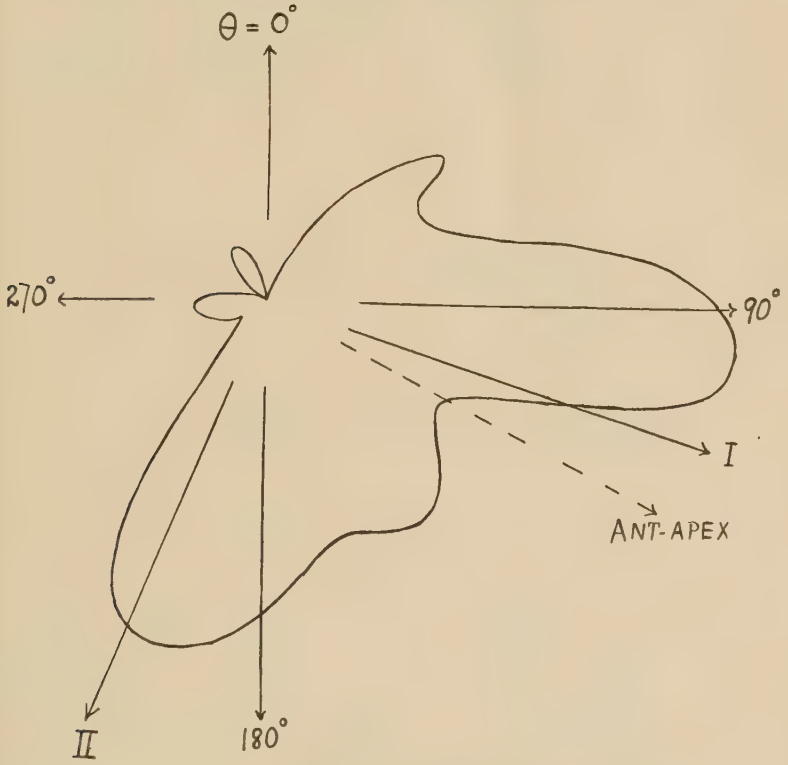


Fig. 2.—β Pegasi.

§ 2. *Region I.* (*Capella*).—With the notation of previous papers, we have for this region :—

Galactic Latitude	$=+6°$.
Position Angle of Ant-Apex	$=169°$.
,, ,, Apex of Drift I.	$=166°$.
,, ,, ,, Drift II.	$=215°$.
Angular Distances λ, λ_I, λ_{II}	$=81°$, $61°$, $157°$ respectively.

Fig. 1 shows the distribution of proper motions of the 181 stars in this region. It should be noticed that $\sin \lambda_{II} \equiv \sin 23°$ is comparatively small : consequently, Drift II. should not be expected to be shown very strongly.

§ 3. *Region II.* (*β Pegasi*).—We have :—

Galactic Latitude $=-29°$.
Position Angle of Ant-Apex $=117°$.
,, ,, Apex of Drift I. $= 96°$.
,, ,, ,, Drift II. $=202°$.
Angular Distances λ, λ_I, λ_{II} $=117°$, $109°$, $102°$ respectively.

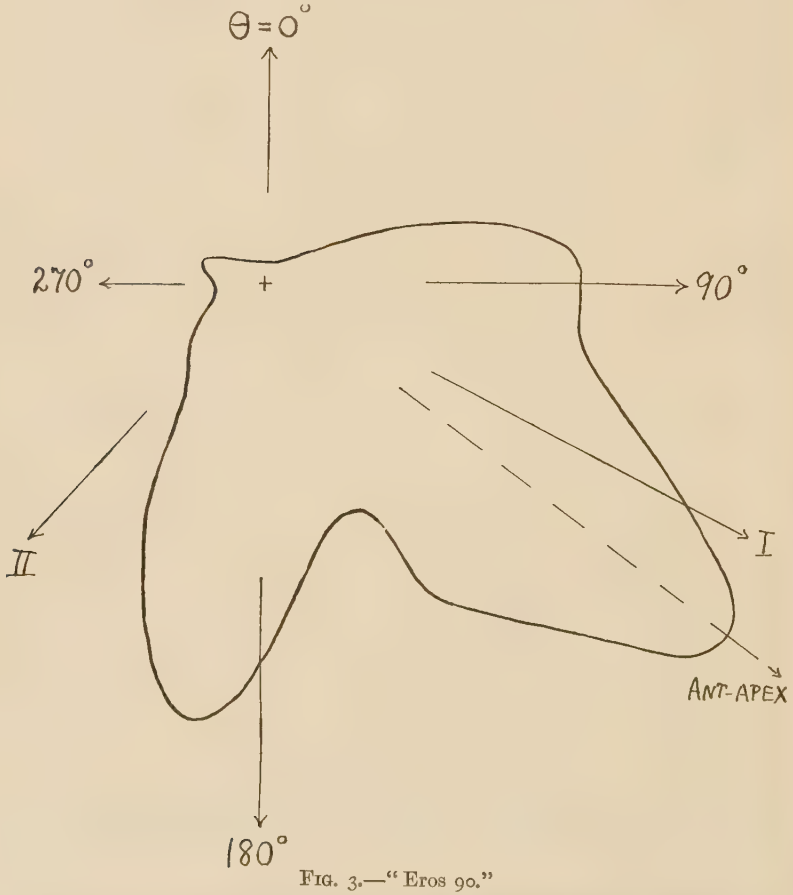


FIG. 3.—"Eros 90."

Fig. 2 shows the distribution of proper motions of the 50 stars. The drift motions are widely separated, and are clearly shown in the figure.

§ 4. *Region III.* ("*Eros* 90").—We have :—

Galactic Latitude $=-7°$.
Position Angle of Ant-Apex $=128°$.
,, ,, Apex of Drift I. $=117°$.
,, ,, ,, Drift II. $=221°$.
Angular Distances λ, λ_I, λ_{II} $=104°$, $87°$, $138°$ respectively.

The distribution of motions of the 61 stars is shown in fig. 3. Here again the drift motions are clearly separated.

§ 5. *Regions IV. and V.* ("*Eros* 102, 103").—For each region we have :—

Galactic Latitude $= -16°$.
Position Angle of Ant-Apex $= 129°$.
" " Apex of Drift I. $= 116°$.
" " " Drift II. $= 215°$.
Angular Distances λ, λ_I, λ_{II} $= 100°, 84°{\cdot}5, 131°{\cdot}5$ respectively.

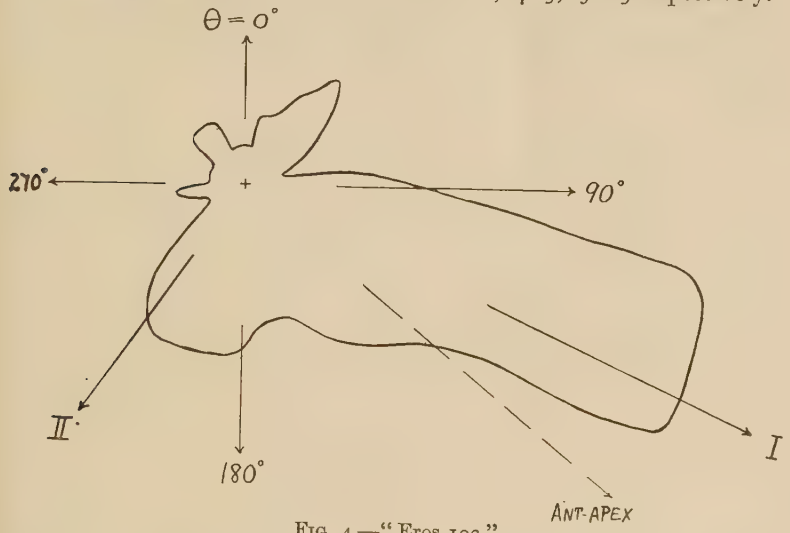


Fig. 4.—"Eros 102."

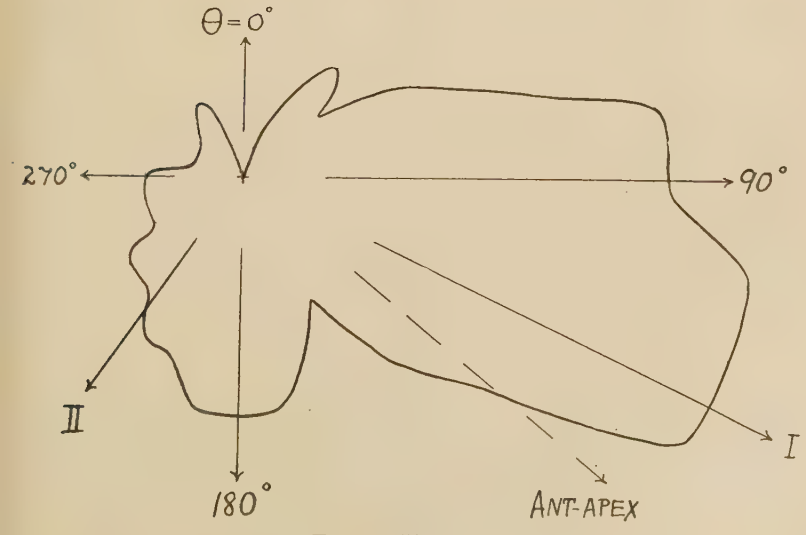


Fig. 5.—"Eros 103."

Figs. 4 and 5 show the distribution of proper motions of Regions IV. and V. respectively. There are 28 stars common to both regions. The directions of the drifts are in good agreement with those calculated, but the strength of Drift II. is feeble in comparison with Drift I. In the combined region there are 5 stars with proper motions between 10″ and 30″ per century.

§ 6. In the following table the derived absolute proper motions of the 28 stars common to Regions IV. and V. are shown. The correction "absolute–relative" is nearly the same in each area. The comparison stars are, in the main, different in the two areas, but the mean magnitudes are much the same. The last two columns give the average discordance in the values of μ_α, μ_δ in the sense "V.—mean of IV. and V."

No.	μ_α.		μ_δ.		ν_α.	ν_δ.
	IV.	V.	IV.	V.		
	″	″	″	″	″	″
1	− 1·7	− 0·7	− 0·6	− 1·4	+0·5	−0·4
2	+ 1·4	+ 0·2	− 0·6	− 1·2	−0·6	−0·3
3	+ 1·8	+ 0·9	− 1·8	− 1·8	−0·5	0·0
4	+ 1·2	+ 0·2	− 0·6	− 0·9	−0·5	−0·1
5	− 0·8	− 1·7	− 1·9	− 2·4	−0·4	−0·3
6	+22·8	+22·8	−22·1	−22·1	0·0	0·0
7	+ 2·7	+ 1·6	− 1·0	− 1·0	−0·6	0·0
8	+ 1·8	+ 0·2	− 0·8	− 1·3	−0·8	−0·3
9	+ 1·2	+ 1·1	+ 0·1	+ 0·2	−0·1	+0·1
10	+ 0·3	+ 0·4	− 0·7	− 0·1	+0·1	+0·2
11	+ 9·0	+ 7·7	− 3·2	− 2·2	−0·7	+0·5
12	+ 1·2	− 0·4	− 0·6	− 1·2	−0·8	−0·3
13	+ 6·3	+ 6·3	− 2·7	− 2·7	0·0	0·0
14	+ 2·1	+ 1·1	− 0·5	− 0·3	−0·5	+0·1
15	+ 3·1	+ 2·3	− 1·6	− 0·6	−0·4	+0·5
16	+ 4·8	+ 3·9	+ 0·2	+ 0·5	−0·5	+0·2
17	− 0·5	− 0·4	− 1·1	+ 0·2	+0·1	+0·7
18	+12·4	+11·0	− 4·4	− 3·8	−0·7	+0·3
19	− 0·3	+ 0·5	− 1·3	+ 0·9	+0·4	+1·1
20	− 1·5	− 1·6	0·0	+ 1·4	0·0	+0·7
21	+ 0·7	− 0·1	− 0·1	+ 0·8	−0·7	+0·5
22	− 0·1	+ 0·1	− 5·1	− 4·9	+0·1	+0·1
23	+ 0·8	+ 0·5	+ 0·3	+ 0·6	−0·2	+0·2
24	+ 3·5	+ 4·0	− 1·2	− 1·2	+0·3	0·0
25	+ 3·1	+ 3·7	− 1·6	− 1·6	+0·3	0·0
26	− 0·1	− 0·1	+ 1·5	+ 1·6	0·0	+0·1
27	− 0·9	− 0·9	+ 0·4	+ 0·1	0·0	−0·2
28	+ 1·2	+ 1·2	− 1·3	− 0·7	0·0	+0·3

The largest discordance is that in declination for star No. 19 on the above list.

In declination, the sum of the positive residuals is +5″·6 and of the negative residuals −1″·9. We could make the algebraic sum vanish by moving, in the appropriate directions—in declination—the origins of IV. and V. (of absolute proper motion) by a quantity about 0″·1 in magnitude. The distributions shown in figs. 4 and 5 would not be sensibly altered on this account.

Parallel to the Equator, the sum of positive residuals ν_α is +1″·8 and of negative residuals −8″·0, and in the same way the algebraic sum would vanish were the origins of IV. and V. shifted—in opposite directions—by 0″·2. Here again figs. 4 and 5 would not show appreciable change.

The average discordances—taken regardless of sign—of ν_α and ν_δ are 0″·35 and 0″·26 respectively.

Seven pairs of images were measured in IV. and six in V. The agreement shown by the above table is satisfactory, especially when one remembers that several images are extremely faint—but just measurable—on the 1900 plates. The probable errors of the means for the 28 stars of the above table are of the order 0″·2 per century. These stars are, on the whole, favourably situated on the plates : the remaining stars, with 6 or 7 comparisons, would give probable errors much larger in magnitude, roughly of the order 0″·6 per century.

Observatory, Cambridge :
1925 March 5.

Spectroscopic Parallaxes of B-type Stars (Third Paper) : Second List of 100 Parallaxes.

By D. L. Edwards, A.R.C.S., D.I.C.

(*Communicated by the Director of the Norman Lockyer Observatory.*)

This paper is the continuation of a previous list of spectroscopic parallaxes * derived from measurements of line intensities in B-type stars by the method therein described. In order to check the accuracy of the reduction curves, some standard stars of known parallax were measured alternately with stars of the present list at frequent intervals, and the results compared with those previously obtained. The differences were so small that no great modification of the reduction curves seemed to be required, and they were used as originally obtained.

Limitations of the Method.

As was remarked in the previous paper on this subject, the fact that the intensities of the Helium lines are used as the basis of the Harvard classification of these stars would seem to imply that this method is mainly a mechanical method of classification, giving (pre-

* *Monthly Notices R.A.S.*, **84**, 366.

sumably) greater refinement than is possible with mere eye estimates. In such a case the determination of parallaxes would depend on the well-known correlation between spectral type and absolute magnitude. It has been found, however, that reduction curves can be founded on stars of a single subdivision of spectral type alone, though, of course, such curves do not extend over the whole range covered by the more general curves founded on all types. For example, it was found that stars of class B3 showed variations in the strength of the Helium lines which could be correlated with corresponding variations in absolute magnitude, giving a curve closely approaching the reduction curves deduced from all types. Also, if stars with the best-determined parallaxes (trigonometrical or group motion) are considered, the residuals from these standards given by the spectroscopic results are considerably smaller than those obtained by assigning fixed absolute magnitudes to the Harvard types.

It would seem, therefore, that these measures give something more than a simple classification of spectral type following the Harvard system. Other quantities are apparently included in the measurements, among which is probably an effect of line characteristics (sharpness or otherwise) which certainly seem to affect intensities as measured by the wedge method.

It has been found by experience, however, that this method does not give good results with certain types of stars. In particular may be mentioned those stars with very broad and diffuse lines. The great breadth and nebulosity of the Hydrogen lines in these stars reduces their apparent intensities with respect to the Helium lines, with the result that the measured absolute magnitudes are usually too bright —though occasionally the reverse is the case, when the Helium lines are more affected than the Hydrogen. In addition, in the case of stars of types B8 and B9 the Helium lines are so faint as to be measurable only with difficulty (in the case of B9 stars they are usually not measurable at all). This uncertainty, however, is compensated for to some extent by the steepness of the reduction curves for these stars. Errors of measurement have only a small effect on the deduced absolute magnitude, and if the lines are measurable at all the resulting parallaxes are only slightly less trustworthy than those for the earlier types.

It appears, therefore, that the very nebulous line stars, together with the majority of the late B-type stars, require to be treated by a different method. That of Adams and Joy, based on line characteristics, appears to be the best spectroscopic method at present available for these stars.

The Table of Results.

In Table I. are given the parallaxes of 100 stars, in continuation of the list previously published. Columns 1 and 2 give the number of the star in the Henry Draper Catalogue, together with the name of the star or the constellation in which it occurs. Columns 3 to 7 give the approximate position of the star for 1900, the visual magnitude, spectral type (H.D.), and total proper motion. Columns 8 and 9 give the spectroscopic absolute magnitude and parallax, while the last two

columns give other parallaxes for comparison together with their authorities. In the column for authorities W=Mount Wilson spectroscopic parallaxes, g=group-motion parallaxes, and t=trigonometrical parallaxes.

The scarcity of trigonometrical parallaxes (given in column 10) makes any detailed comparison with the present results impracticable. With regard to group-motion parallaxes, the agreement is not so good as it was in the previous list. Twenty-two stars given here have group-motion parallaxes, the mean of which is 0″·009. The corresponding spectroscopic mean value is 0″·006.

The systematic difference between the results obtained here and those of Adams and Joy, which was very marked in the first list, is still evident, though its actual value (0·4 of a magnitude) is not so large as that previously obtained. The difference is possibly due to different standard parallaxes adopted in the two methods, but no correction has been applied to these results, since a general comparison of spectroscopic values from this and the previous list, with the trigonometrical and group-motion parallaxes taken together, does not give a definite indication of systematic error.

TABLE I.

1. H.D.	2. Name.	3. 1900. R.A.	4. 1900. Dec.	5. *m.*	6. Spectrum (H.D.)	7. μ	8. N.L.O. Spectroscopic. M.	9. N.L.O. Spectroscopic. π.	10. Other. Parallaxes. π.	11. Other. Parallaxes. Auth.
		h m	° ′			″		″		
1337	– Cas.	0 12·5	+50 53	6·1	B0	0·011	−2·5	0·002		
1606	– And.	15·2	+30 23	5·8	B8	..	−0·8	·005		
1976	– Cas.	18·9	+51 28	5·4	B3	·017	−1·0	·005	·006	W.
2729	13 Cas.	25·7	+65 58	6·1	B5	·031	−0·6	·005		
3240	– Cas.	30·5	+53 38	5·1	B5	·021	0·0	·009	·010	W.
3369	π And.	31·5	+33 10	4·4	B3	·023	−1·2	·008	−·008	t.
4142	– Cas.	38·8	+47 19	5·6	B3	·026	−1·2	·004		
4180	o Cas.	39·2	+47 44	4·7	B2	·023	−1·8	·005		
6300	– Cas.	58·9	+50 29	6·5	B3	..	−1·9	·002		
11241	1 Per.	1 45·4	+54 39	5·5	B3	·035	−1·9	·003	·005 ·009	W. g.
11529	ω Cas.	48·2	+68 12	5·0	B8	·010	−0·7	·007	·008	W.
12303	4 Per.	55·6	+54 1	5·0	B8	·042	0·0	·010	·011	W.
14228	ϕ Eri.	2 12·9	−51 59	3·8	B8	·043	−0·3	·015		
16727	11 Per.	35·8	+54 41	5·7	B8	·057	−0·1	·007	·014	g.
17543	π Ari.	43·7	+17 3	5·3	B5	·022	−1·2	·005	·007	W.
17769	σ Ari.	46·0	+14 40	5·5	B5	·045	−0·5	·006	·007	W.
18604	λ Cet.	54·4	+ 8 31	4·7	B5	·011	−0·3	·010		
20365	29 Per.	3 11·5	+49 51	5·3	B3	·044	−1·6	·004	·006 ·011	W. g.

TABLE I.—*continued.*

1. H.D.	2. Name.	3. 1900. R.A.	4. 1900. Dec.	5. *m.*	6. Spectrum (H.D.)	7. μ	8. N.L.O. Spectroscopic. M.	9. N.L.O. Spectroscopic. π.	10. Other Parallaxes. π.	11. Other Parallaxes. Auth.
		h m	° ′			″		″		
20418	31 Per.	3 12·0	+49 43	5·1	B3	0·041	−1·0	0·006	·007 ·010	W. g.
20756	τ Ari.	15·5	+20 47	5·2	B3	·042	−1·5	·005		
21071	– Per.	18·9	+48 45	5·9	B5	..	−0·6	·005	·007 ·008	W. g.
21278	– Per.	20·9	+48 43	4·9	B5	·032	−0·9	·007	·008	g.
23466	29 Tau.	40·3	+ 5 44	5·4	B3	·030	−1·3	·005		
27192	– Per.	4 12·6	+50 41	5·5	B3	·015	−1·7	·004	·006	W.
29335	49 Eri.	32·1	+ 0 48	5·3	B5	·018	−0·6	·007	·007	W.
32249	ψ Eri.	56·6	− 7 20	4·8	B8	·010	−2·0	·004	·006	g.
34503	τ Ori.	5 12·8	− 6 57	3·7	B5	·017	−1·1	·011	·013 ·007	W. g.
36512	υ Ori.	27·1	− 7 23	4·6	B3	·012	−2·0	·005	·004 ·006	W. g.
37490	ω Ori.	33·9	+ 4 4	4·5	B3*p*	·006	−2·3	·004	·005	g.
37711	126 Tau.	35·5	+16 29	4·9	B3	·031	−1·8	·005	+·007	t.
41753	ν Ori.	6 1·9	+14 47	4·4	B2	·037	−2·4	·004		
42545	69 Ori.	6·2	+16 9	4·9	B3	·024	−1·2	·006	·010	g.
42690	– Mon.	7·0	− 6 31	5·1	B3	·013	−1·9	·004	·008	g.
44112	7 Mon.	14·9	− 7 47	5·1	B3	·008	−1·7	·004	·008	g.
45542	ν Gem.	23·0	+20 17	4·1	B5	·023	−0·9	·010	·009	g.
45546	10 Mon.	23·0	− 4 42	5·0	B3	·014	−1·8	·004	·006	g.
46487	– Mon.	28·6	− 1 9	5·0	B3	·051	−0·8	·007		
48879	42 Cam.	40·5	+67 41	5·0	B3	·009	−1·3	·006		
52918	19 Mon.	57·9	− 4 6	4·9	B3	·008	−1·4	·006	·005	g.
67797	– Pup.	8 4·5	−18 57	4·3	B3	·016	−1·4	·007	·006	g.
109668	α Mus.	12 31·2	−68 35	2·9	B3	·041	−2·0	·011	·014	g.
120307	ν Cen.	13 43·5	−41 11	3·5	B2	·042	−2·5	·006	·011	g.
121263	ζ Cen.	49·3	−46 48	3·1	B2*p*	·082	−2·0	·009		
149757	ζ Oph.	16 31·7	−10 22	2·7	Bo	·021	−2·5	·009		
154445	– Oph.	17 0·4	− 0 45	5·6	B3	·008	−1·5	·004	·005	W.
155763	ζ Dra.	8·5	+65 50	3·2	B5	·024	−0·7	·016	·014 ·019	W. t.
156633	68 Her.	13·6	+33 12	var.	B3	·024	−1·8	·005	·005	dyn.
163472	– Oph.	51·2	+ 0 42	5·7	B3	·006	−2·0	·003	·004	W.
166182	102 Her.	18 4·4	+20 48	4·3	B3	·017	−2·3	·005		
167965	– Lyr.	12·5	+42 8	5·4	B5	·009	−0·1	·008	·008	W.

TABLE I.—*continued.*

1. H.D.	2. Name.	3. 1900. R.A.	4. 1900. Dec.	5. *m.*	6. Spectrum (H.D.)	7. μ	8. N.L.O. Spectroscopic. M.	9. N.L.O. Spectroscopic. π.	10. Other. Parallaxes π.	11. Other. Parallaxes Auth.
		h m	° ′			″		″		
169467	α Tel.	18 19·6	−46 1	3·8	B3	0·053	−1·6	0·008	·013	g.
171301	− Lyr.	29·0	+30 29	5·4	B8	·007	−0·3	·007	·008	W.
173300	ϕ Sgr.	39·4	−27 6	3·3	B8	·048	−0·7	·016		
174179	− Lyr.	44·2	+31 39	5·8	B3	..	−1·5	·004		
175156	− Sct.	49·0	−15 43	5·0	B5	·023	−1·2	·006	·006	W.
175191	σ Sgr.	49·1	−26 25	2·1	B3	·067	−2·2	·014	·018	g.
175426	δ^1 Lyr.	50·2	+36 51	5·5	B3	·012	−1·7	·004	·005	W.
177003	− Dra.	57·7	+50 24	5·2	B3	·025	−2·0	·004	·005	W.
178475	ι Lyr.	19 3·7	+35 57	5·1	B5	·009	−0·6	·007	·008	W.
180163	η Lyr.	10·4	+38 58	4·5	B3	·003	−2·2	·005		
180554	1 Vul.	11·9	+21 13	4·6	B5	·004	−1·2	·007		
182255	3 Vul.	18·7	+26 4	4·9	B5	·015	−0·7	·008	·009	W.
182568	2 Cyg.	20·2	+29 26	4·9	B3	·014	−2·0	·004		
184171	8 Cyg.	28·1	+34 14	4·9	B3	·004	−1·7	·005		
184930	ι Aql.	31·6	− 1 31	4·3	B5	·018	−0·9	·009		
187811	12 Vul.	46·8	+22 21	4·9	B3	·029	−1·3	·006		
188209	− Cyg.	49·0	+46 47	5·5	Bo	·015	−2·9	·002	·002 +·009	W. t.
188892	22 Cyg.	52·3	+38 13	4·9	B3	·012	−1·4	·006	·006	W.
189178	− Cyg.	53·7	+40 6	5·4	B3	·024	−0·9	·006	·007	W.
189687	25 Cyg.	56·2	+36 46	5·2	B3	·011	−1·3	·005		
190993	17 Vul.	20 2·5	+23 19	5·1	B3	·008	−1·5	·005	·007	W.
192685	− Vul.	11·0	+25 17	4·8	B3	·003	−1·3	·006		
193924	α Pav.	17·7	−57 3	2·1	B3	·086	−2·2	·014		
195556	ω^1 Cyg.	27·0	+48 37	4·9	B3	·010	−1·3	·006		
196740	28 Vul.	34·2	+23 46	5·0	B5	·011	−0·3	·009		
197511	51 Cyg.	39·1	+49 59	5·4	B3	·006	−1·5	·004	·005	W.
198478	55 Cyg.	45·5	+45 45	4·9	B2	·005	−2·4	·004		
199081	57 Cyg.	49·7	+44 0	4·7	B3	·018	−0·9	·008		
200120	59 Cyg.	56·4	+47 8	4·9	B*op*	·008	−2·5	·003		
200310	60 Cyg.	57·6	+45 46	5·2	B3	·004	−0·9	·006	·008	W.
204172	69 Cyg.	21 21·7	+36 14	5·8	Bo	·011	−3·0	·002	·002	W.
204403	70 Cyg.	23·2	+36 41	5·2	B3	·004	−1·7	·004	·006	W.
205637	ϵ Cap.	31·5	−19 54	4·7	B5*p*	·007	−0·9	·008		
206165	9 Cep.	35·2	+61 38	4·9	B2*p*	·005	−2·3	·004		
206672	π^1 Cyg.	38·6	+50 44	4·8	B3	·006	−1·2	·006		
207330	π^2 Cyg.	43·1	+48 51	4·3	B3	·005	−2·3	·005		

TABLE I.—*continued.*

1. H.D.	2. Name.	3. 1900. R.A.	4. 1900. Dec.	5. *m.*	6. Spectrum (H.D.)	7. μ	8. N.L.O. Spectroscopic. M.	9. N.L.O. Spectroscopic. π.	10. Other Parallaxes. π.	11. Other Parallaxes. Auth.
		h m	° ′			″		″		
208057	16 Peg.	21 48·5	+25 27	5·1	B3	0·002	−1·5	0·005	·007	W.
209008	18 Peg.	55·1	+ 6 14	6·0	B3	·006	−1·2	·004	·004	W.
211924	30 Peg.	22 15·5	+ 5 17	5·4	B5	·019	−0·9	·006	·005	W.
213420	6 Lac.	26·1	+42 36	4·5	B3	·013	−1·7	·006	·007	W.
214993	12 Lac.	37·0	+39 43	5·2	B2	·021	−2·2	·003	·004	W.
216200	14 Lac.	45·8	+41 26	5·8	B5	·001	−0·8	·005		
216916	16 Lac.	51·9	+41 4	5·5	B3	·008	−1·9	·003		
217050	– Lac.	52·7	+48 9	5·2	B3	·022	−1·1	·006		
217101	– Lac.	53·1	+38 48	6·1	B3	..	−1·8	·003		
218376	1 Cas.	23 2·4	+58 53	4·9	B1	·012	−2·7	·003		
218440	– Cas.	3·0	+59 13	6·3	B3	..	−1·5	·003		
221253	– Cas.	25·4	+58 0	4·9	B3	·028	−1·8	·005		
223128	– Cep.	41·8	+66 13	5·9	B3	..	−2·0	·003		
224572	σ Cas.	53·9	+55 12	4·9	B2	·012	−2·2	·004		

Summary.

This list of parallaxes is the continuation of a previous list published in the *Monthly Notices*, **84**, 366. The same method has been used in obtaining them, and the original reduction curves have not been found to require any modification.

The method has been found to break down in the case of stars with very diffuse lines, and also in the case of late B-type stars in which the Helium lines are not sufficiently strong to be measurable with the wedge.

Note on Spectroscopic Parallaxes. By Rev. H. Macklin, S.J., B.Sc.

(*Communicated by Rev. A. L. Cortie, S.J.*)

An important branch of the work done at the Stonyhurst College Observatory has been the photography of stellar spectra, particularly of new stars, and of stars possessing peculiar spectra. However, a sufficient number of the more normal stellar spectra had been photographed to justify an examination of them for the purpose of obtaining spectroscopic parallaxes. But as only stars not fainter than about the third or fourth magnitude gave spectra intense enough for this purpose, there is not sufficient material as yet available for more than a preliminary examination, although more photographs are being taken, which will be used later.

The spectrograms are of two kinds: those taken with the 4-inch

Cooke equatorial and Thorp objective prism, and those taken with the Hilger spectroscope attached to the 15-inch equatorial. In both cases the dispersion is small, with the objective-prism camera 14·26 mm. from H_β to H_ϵ, and with the Hilger prism 10·71 mm. for the same extent of spectrum.

For estimating the intensities of the lines in the spectra a photographic wedge, fitted to the Hilger measuring machine, was employed, which was kindly presented to the observatory by Dr. Hemsalech. It is a replica of an excellent Cooke wedge. The method used was simply to measure the number of millimetres the wedge had to be moved from a given fixed point to give total extinction of the line under examination.

On account of the comparative smallness of the dispersion, the close pairs of lines as used at the Mount Wilson Observatory could not be employed, so that experiments had to be undertaken to find pairs sufficiently separated and sufficiently distinct to be seen on plates of a fairly wide range of spectral type.

First of all seven stars of types G8 to K4 were examined, using the pair λλ4455 and 4215; and later the line λ4326 was added as a standard line; however, the results were unsatisfactory.

A more thorough examination of the same class of stars was made, using the same pairs, but no satisfactory curve was obtained connecting these differences with the absolute magnitudes published by the Mount Wilson Observatory in No. 199 of *Contributions from the Mount Wilson Observatory*, which were taken throughout as the standard absolute magnitudes for the stars examined. The failure of this attempt was probably due to the very short range of absolute magnitudes available —less than three magnitudes. More stars were available of types F7 to G7, so these were next examined, using the pairs λλ4326 and 4215, λλ4326 and 4290. The results here were better but not very consistent, so a more extended scheme was carried out for the same class.

Two plates of stars of the same spectral type, whose absolute magnitudes differed by about five magnitudes, were placed side by side under the microscope, so that the corresponding lines of the spectra exactly adjoined; those lines were chosen which changed most noticeably in intensity with this change of absolute magnitude. Of these λλ4227 and 4326 decreased slightly in intensity with the increase of absolute brightness; the others, viz. λλ4174, 4290, 4387, 4444, increased notably with the increase of absolute brightness.

So the pairs λλ4227, 4174, λλ4326, 4290, λλ4326, 4387, and λλ4326, 4444 were employed for the spectra of fourteen stars in the above class.

The results were much better, though no pair gave complete satisfaction. So a search was made for all possible causes that might be responsible for the discordant results.

First of all there was the question of unequal illumination, as hitherto daylight had been used for illuminating the plate under examination. For this electric light was now substituted, dispersed by a screen so as to give uniform illumination; also the plate was screened from any stray daylight.

Secondly, it was found that where the intensities were high, on

account of the great darkness of the field required before the wedge photometer extinguished the line under examination, the intensities were made too great, probably through eye-weariness and other subjective causes. After various corrections had been tried, following on a series of experiments, it was found most satisfactory to omit all fainter negatives, which gave high intensity values; or if there were no other plates available, on account of the faintness of the star, to dim the plate by a screen, so as to make the intensities more normal. This procedure gave concordant results in the very few cases where it had to be employed.

A new set of measures was now made, commencing with stars in the F7 to G7 classes. The line λ4174 was omitted, as the spectrum was often too weak in this neighbourhood, and other lines were added, so that finally the following lines were used: λλ4227 Ca, 4326 Fe, 4215 Sr^+, 4290 Ti^+, 4353 V^+, 4387 Ti^+, 4444 Ti^+—of which the first two decrease in intensity with increase of absolute magnitude, and the other five increase under the same conditions.

The constant for each star was obtained by subtracting the mean of the intensities of the last five lines from the mean of the intensities of the first two. It was hoped that by this method much more accurate results might be obtained.

For, firstly, λλ4227 and 4326 are affected in opposite directions by the advance of spectral type, and so their mean might be expected to reduce to a very small quantity any effect produced by this cause. This proved to be correct, so much so that the same curve was used for a very much wider range of spectral types and still gave very concordant results.

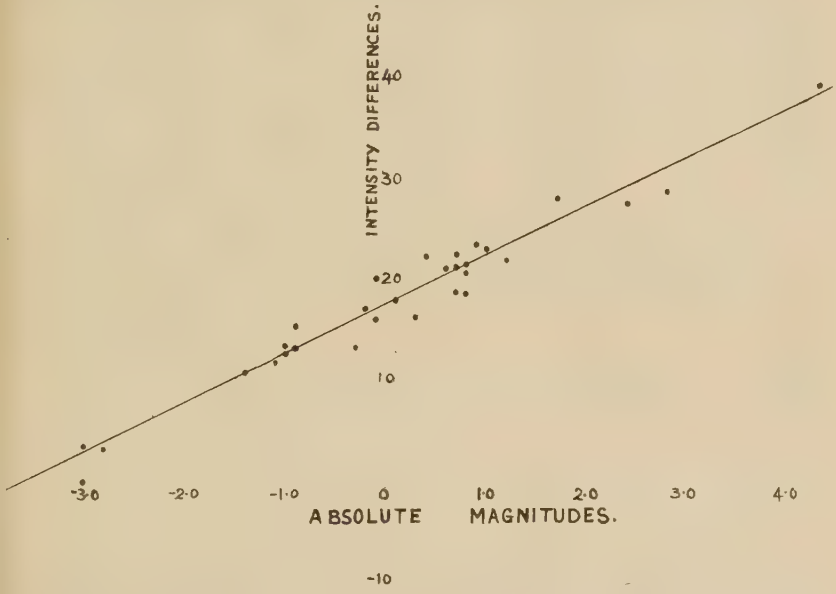
Secondly, the taking of the mean of intensities of a number of lines tended to eliminate the casual errors of measurement, while giving a wide range of values for intensity differences.

This method was then employed on all plates of the F, G, and K types of spectra available, each star having values from at least two plates, some from as many as five or six. The intensity of each line was measured three times on each plate. Also, in the later stages, Father Cortie kindly took independent readings of the intensities of lines, so as to check any errors that might be due to the observer.

Altogether thirty-seven stars were examined in this way, including all stars of those groups whose spectra were available at the Stonyhurst Observatory. However, it was discovered that the earlier F-type stars did not fit the curve, owing to the very notable influence of spectral type at this stage, especially on the line λ4326. The others, including stars from the types F8 to K3, fitted the curve well; and the results are shown below in a graph in which absolute magnitudes (from the Mount Wilson catalogue) are plotted against the intensity difference for each individual star.

Finally, the results are grouped in tabular form as follows:—The first column gives the name of the star, the second its spectral type according to the Mount Wilson scheme, the third the Harvard visual magnitude, the fourth the intensity difference for each star as determined by our measures, the fifth the absolute magnitude calculated

from the graph, the sixth the parallax hence obtained, and the seventh the spectroscopic parallax from the Mount Wilson tables. The eighth and ninth columns have been added from the results published from the



Norman Lockyer Observatory by Mr. Rimmer (*Memoirs R.A.S.*, **62**, 1923; **64**, 1925).

Star.	Type M.W.	Visual Mag. Harv.	δ.	Calc. Abs. Mag.	π. Stony-hurst.	π. M.W.	π. N.L.O.	Trig.	
					″				
α Cas.	G3	2·5	13·3	−0·8	0·022	0·020	0·018	0·016	M.Ye.
η Cas. B	F8	3·7	39·9	4·5	0·144	0·132	0·138	0·183	A.M.S.
α UMi.	F8	2·1	3·1	−2·9	0·010	0·010	0·014	0·007	G.F. 0·016 Ceph.
γ And.	Ko	2·3	20·2	0·5	0·044	0·033	0·040	0·015	A.S.M.
α Ari.	Ko	2·2	18·7	0·2	0·040	0·050	0·048	0·034	A.
γ Per.	F9	3·1	16·0	−0·3	0·021	0·023	0·025	0·011	A.M.
α Tau.	K3	1·1	23·1	1·1	0·100	0·096	0·077	0·052	M.W.Ye.
α Aur.	G1	0·7	17·8	0·1	0·076	0·076	0·076	0·076	A.Ye.
ε Gem.	G5	3·2	10·5	−1·4	0·012	0·012	0·011	0·009	A.M.
κ Gem.	G6	3·7	23·5	1·2	0·032	0·028	0·023	0·024	A.M.
β Gem.	G8	1·2	28·3	2·2	0·159	0·126	0·105		
ε Leo.	Go	3·1	15·3	−0·4	0·020	0·016	0·016		
γ Leo. B	G8	2·6	21·6	0·8	0·044	0·044	0·045	0·006	A.M.
μ UMa.	K1	3·2	21·3	0·8	0·033	0·032	0·034		

Star.	Type M.W.	Visual Mag. Harv.	δ.	Calc. Abs. Mag.	π. Stony-hurst.	π. M.W.	π. N.L.O.	Trig.	
					″				
α UMa.	G8	2·0	22·3	1·0	0·063	0·048	0·050		
η Boo.	G0	2·8	28·9	2·3	0·079	0·100	0·087		
α Boo.	G9	0·2	22·0	0·9	0·135	0·158	0·145	0·100	Ye.
ε Boo.	G7	2·7	13·1	−0·9	0·019	0·019	0·020	0·014	A.
η Dra.	G5	2·9	22·5	1·0	0·042	0·036	0·030	0·007	M.
β Her.	G6	2·8	12·3	−1·0	0·017	0·017	0·020	0·036	S.
ζ Her.	G1	3·0	27·7	2·0	0·063	0·076	0·100	0·103	A.S.Ye.
γ Dra.	K3	2·4	21·1	0·7	0·046	0·044	0·038	0·017	A.M.F.
δ Dra.	G7	3·2	18·5	0·2	0·025	0·033	0·032	0·035	A.
γ Aql.	K1	2·8	16·2	−0·3	0·024	0·032	0·033	0·020	A.M.
γ Cyg.	G0	2·3	− 0·5	−3·6	0·007	0·009	0·008	−0·002	A.M.F.
ε Cyg.	G7	2·6	20·6	0·6	0·040	0·044	0·048	0·037	A.M.Ye.
ζ Cyg.	G7	3·4	11·6	−1·2	0·012	0·013	0·015	0·026	A.M.
ε Peg.	G7	2·5	13·1	−0·9	0·021	0·028	0·032	0·003	A.M.
α Aqr.	F9	3·2	2·8	−2·9	0·006	0·006	0·007	0·009	M.
η Peg.	G2	3·1	17·1	−0·1	0·023	0·022	0·019	0·000	A.M.S.

Dr. Lockyer very kindly lent a number of plates with which to supplement this work, but, unfortunately, these plates were so entirely different in dispersion from the Stonyhurst plates that it was impossible to use them in combination with our own photographs. Consequently, as it might be some time before the number of Stonyhurst plates available would be largely increased, it was thought better to give the results already obtained, as an illustration of a method adopted with spectra of small dispersion, though they extend to such a comparatively small number of stars.

Comparing the results with those from Mount Wilson and from the Norman Lockyer Observatory, independent of sign, the mean difference between Stonyhurst and Mount Wilson is ″·006, and between Stonyhurst and the Norman Lockyer Observatory ″·009. In the case of Capella the three results are identical among themselves and agree exactly with the trigonometrical parallax. The most divergent results are from Pollux. In general the agreement is best in stars of great absolute magnitude, independent of spectral type.

Stonyhurst College Observatory :
1925 *March* 5.

The Secondary Spectrum of Hydrogen. By J. W. Nicholson, F.R.S., D.Sc., Fellow of Balliol College, Oxford.

On many occasions, the writer has advanced the view that the key to the further extension of the quantum theory of spectra lies in the secondary spectrum of hydrogen, rather than in the emission spectra of atoms, and even of helium. The present communication, which deals with work done some time ago, is intended to deal with certain specific problems presented by this spectrum, but does not claim to be in any way exhaustive.

There is no doubt that the spectrum has many component parts—due, for example, to neutral hydrogen molecules, ionised molecules, molecular aggregations such as H_3, negatively charged atoms, and so forth—and that these are all of different types, presenting a total complex of line-spectra and of elongated band-spectra whose nature it is difficult to elucidate.

Many investigators have now taken up this subject, confining attention to certain parts of the spectrum which seem to belong together, and it is generally recognised that the spectrum consists of at least two parts.

There is a considerable spectrum beyond the limit of the Balmer series towards the ultra-violet. It is probable that many of the lines thus concerned belong, in fact, to the spectrum of water-vapour, but the bulk of the spectrum cannot be regarded as arising from such a source. The original discovery by Goldstein and Curtis of a band-spectrum associated with helium was in itself an effective suggestion that a similar spectrum should exist in hydrogen, and it is natural to expect its occurrence as a component part of the secondary spectrum.

Dufour's analysis of the spectrum, in the light of the Zeeman effect, gave the first indication of the existence of two distinct types of lines, and possibly even of a third type. Although only a selection from the strongest lines was submitted to examination, some showed a strong Zeeman effect, and others none at all, while a not numerous third type of line seemed to be anomalous in behaviour. Dufour stated that some of the strongest lines appeared to show constant frequency differences, but he gave no numerical results in this connection; others, for example Paulsen, have made similar suggestions, but the instances they give do not appear to be satisfactory and beyond the bounds of coincidence in such a closely-packed spectrum, and, moreover, they include lines which do, and which do not, show a normal Zeeman effect. The writer is unable to consider that these suggestions are of value towards the interpretation of the spectrum, or towards its arrangement according to definite laws.

The next step in the elucidation of the secondary spectrum was taken by Fulcher, who showed that a set of lines in the red, which repeat themselves at intervals of about 100 Å., are the strongest lines in the low-potential discharge through hydrogen, a fact which tempts us to make the suggestion at once that they are to be expected to be due to the neutral or un-ionised hydrogen molecule. They can indeed

be made very prominent in the laboratory, under such conditions, and much stronger than the Balmer series, which fact, in view of the Bohr theory, is itself sufficient to show that most of the hydrogen present in such experiments is molecular rather than atomic.

These groups of lines were regarded by Fulcher as "bands," although their mutual separations are much larger than those in any known band-spectrum, being in fact of a quite different order of magnitude. But there is no evident reason why scattered bands should not exist and more especially for the very light elements, and the complete examination of such a spectrum might be expected to throw light on the real nature of the general band-spectrum formula, which, quite apart from any current theoretical suppositions arising from the advent of the quantum theory, is not always that of Deslandres, although the latter describes so many bands very completely.

The present communication proposes to show that the Fulcher bands, as they will be called, are a single set, and that the two examples isolated by Fulcher have a significant numerical relation. As they are related, they must in future be regarded as two parts of a single band-system, whose structure shows great complexity.

Several recent writers have considered these bands in particular, but without reaching the conclusions to which, on this aspect of the matter, we are led in the present communication. Merton and Barratt,* in the Bakerian lecture of 1922, concluded that they were essentially a low-pressure spectrum, in which the intrinsic intensity of the light is very small and the walls of the tube are very strongly phosphorescent. Hughes and Lowe † indicated the rapid decline of intensity with increasing voltage.

As to the origin of the Fulcher bands, this paper does not propose to be explicit, but reference must be made to papers by Kiuti ‡ and Allen.§ Kiuti worked at a pressure of about 0·6 of an atmosphere, and found a spectrum which was in many respects different from that given by the ordinary discharge, and found a peculiar spectrum of the type

$$\nu = a + bm^2$$

in the orange region. He considered this to be a Q-series, and that the positive and negative branches of the series were provided by the Fulcher bands. There are many difficulties inherent in this interpretation, which are commented upon by Allen, but, in our opinion, the conclusive argument is provided by the very different conditions under which the spectra are prominent. In the arc at high pressure, as shown in the photographs in his paper, the new series is a strong one, but the Fulcher lines (or bands) have an abnormally low intensity. We are compelled to accord with Allen's conclusion that the experimental evidence, at present, is against the immediate possibility of associating other lines in the secondary spectrum of hydrogen with the Fulcher bands. The considerations we wish to advance at present have a certain

* *Phil. Trans.* A, **222**, 369 *et seq.*
† *Physical Review*, **21**, 292 (1923).
‡ *Proc. Phys. Math. Soc. Japan*, **5**, 9 (1923).
§ *Proc. Roy. Soc.*, A, **106**, 69 (1924).

fundamental interest which is not touched upon in the papers which may be cited at the moment—all based on a specific view of the nature of molecular spectra, which at present is on trial, and subject to various alternative suppositions. We would prefer to discuss the spectrum as it stands, without reference to any definite theory. Interpretations based on the quantum theory are still vague, in that they depend on a conception of the "moment of inertia" of a molecule which—at least in the case of light molecules such as that of hydrogen—can have no definite mathematical meaning.

In the present paper we shall, to a great extent, restrict our attention to a view of the Fulcher bands which was developed some time ago. Fulcher contended that these band-lines were those which showed no Zeeman effect, but he only examined those, in the light of Dufour's work, on the red side of λ 5303. In the following table, a comparison of Fulcher and Dufour's results is made. Z indicates the presence of a normal Zeeman effect, N no observable Zeeman effect. The table is designed to show that lines becoming stronger in the low potential discharge, and therefore likely to belong to the present band system, are essentially those without a Zeeman effect.

In relation to the question of Zeeman effect, some useful remarks may be made at this point. It does not appear that any definite suggestion has yet been put forward as to the reason why band-lines so frequently fail to show the effect. According alike to the quantum theory and the older theories of the Zeeman effect, a *single* electron has always been supposed to be the emitting centre of the affected line, the perturbations of its motions, or, in the quantum theory, the changes of its stationary states—which become more numerous in the presence of a magnetic field—being responsible for the effect. Absence of the effect may mean one of two things, or both, and no further alternative seems possible. In molecular spectra, to which bands are with much reason relegated entirely for their origin, the nuclei may take part in the motion to a great extent, and their degrees of freedom must then be quantised by phase integrals. The enormous (relative) mass of a nucleus would reduce the effect to be expected to an order not capable of observation (in a ratio m/M, as regards order). Secondly, it may be that there is no *single* electron constituting the emission-centre, but two or more electrons moving alike, as in a ring, and it is possible that rings in molecules are at least much more probable stationary states than they can be in atoms.

On these considerations, it seems probable that lines without a measurable Zeeman effect can only be due to molecules, but the converse is not necessarily true, for it is clearly possible to have molecules in which one electron is much more remote than the others. In a general way, we can in this manner envisage the possibility of the whole secondary spectrum being in fact of molecular origin, and the general nature of the molecular stationary states corresponding to each class of line is clear. For the Fulcher bands we should expect *à priori* that the molecular states were such that considerable movement occurred in the nuclei during a change from one state to another, and perhaps also that the two electrons of a hydrogen molecule emitting these bands were in

a ring formation, or at least a formation in which the average distances of the electrons were comparable. Such considerations should form a valuable guide towards the selection of stationary states, in other systems than hydrogen, capable of providing an interpretation of the known spectra of these systems, provided that a more complete analysis of the spectra in regard to Zeeman effect is carried out. At present experimental material of this type is very scanty, and there is an urgent need for Dufour's work to be extended more fully even over the hydrogen spectrum.

We proceed now to give the table of comparison of Fulcher's lines and Dufour's.* The lines are in two groups. Group I. consists of lines whose intensity is not appreciably altered in the high or low pressure spectrum.

GROUP I.

	λ.	Int. Low.	Int. High.		λ.	Int. Low.	Int. High.
	6441·83	½	½	N	6127·53	3	4
	6428·36	3	3	N	6122·01	12	12
	6399·71	4	3	N	6096·21	6	6
	6362·67	2½	2	N	6080·03	7	8
	6340·75	2½	2	Z	6070·21	} 4	} 6
	6327·25	9	9	N	6067·96		
	6299·60	6	5	N	6041·27	1	1½
	6285·56	3	2	N	6032·14	} 7	} 12
	6271·55	1	1	Z	6028·21		
N	6238·57	5	5	N	6018·53	16	14
	6230·48	1	1	N	5994·30	2	2½
N	6225·02	11	11	N	5989·47	2	3
N	6199·58	} 7	} 10	N	5975·68	10	10
N	6197·32			N	5950·15	5	5
N	6183·19	5	4	N	5925·09	3	2
N	6135·56	8	11	N	5916·76?	1	5?

It is clear that the only apparent exceptions in this group are cases of doublets, one component showing Zeeman effect and not the other. These doublets were seen by Fulcher as broad single lines, and not separately. The doubtful case, λ 5916·76, is in the same category and is probably not a single line.

Group II. contains lines which are definitely enhanced in the high potential discharge. It is difficult, on the above basis, to avoid the conclusion that they correspond to an ionised molecule; such molecules are known to occur in large numbers under such conditions in the tube from experiments on positive rays. This molecule would only possess a single electron, so that there is every probability of the occurrence of a normal Zeeman effect.

* For further comparison, see Merton and Barratt (*loc. cit.*). The conclusions are identical.

GROUP II.

	λ.	Int. Low.	Int. High.		λ	Int. Low.	Int. High.
	6380·30	0	1	Z	5931·62	3	9
	6275·10	0	3	N	(5916·76)?	1	5
	6174·28	0	4	Z	5779·22	½	3
	6161·81	½	4		5759·76	2	7
Z	6091·16	0	5		5713·59	1	4
Z	6070·21	} 4	} 6		5703·50	1	8
N	6067·96				5689·00	} 3	} 10
N	6032·14	} 7	} 12		5688·00		
Z	6028·21				5684·33	2	10
Z	6003·08	½	4		5642·65	0	3
Z	5982·79	½	6		5635·05	½	4
Z	5963·70	} 2	} 5		5612·77	½	5
Z	5959·98				5499·84	1	3
Z	5938·87	3	9				

There is again no exception, beyond the doublets already indicated, which are inserted in both tables. For convenience, we now give the remainder of Group I. down to λ 5303·3, Fulcher's limiting band line, though only 5 of this set have been examined by Dufour.

GROUP I.—*continued.*

	λ.	Int. Low.	Int. High.	λ.	Int. Low.	Int. High.
	5909·71	1	1½	5597·80	3	3
	5872·12	} 5	} 5	5552·60	3	2
	5869·47			5543·71	1½	2
N	5836·28	7	10	5537·67	10	8
N	5822·99	5	6	5519·12	1	1
N	5786·00?	7	10	5505·79	4	4
N	5775·28	6	8	5435·06	2	3
N	5737·06	6	9	5426·20	3	4
	5671·10	1½	2	5420·10	7	7
	5655·98	3	5	5388·35	4	4
	5600·65	3	3	5303·35	4	4

We have thus extended Fulcher's conclusion on experimental grounds, that the band lines show no Zeeman effect, to some general considerations leading to the probable origin of both sets of lines. One or two further remarks are necessary, for Fabry and Buisson appeared to have shown at one time, by the method of limiting interference, that some of the lines at least had an atomic origin. Merton and Nicholson's investigation (*Phil. Trans.*, 1918) shows that such conclusions must be accepted with caution in view of the other processes of broadening to which spectral lines are subject. They showed, for instance, that the

conditions under which the shape of the energy curve of a line is of the form

$$I = Ae^{-kx^2}$$

—where x is on the wave-length scale—necessary for Rayleigh and Schönrock's theoretical investigation on which the Buisson and Fabry method is founded, can be defined very sharply. In particular, if the discharge tends to approximate to the so-called "condensed discharge," the lines are split into the components peculiar to the Stark effect, each component being broadened not according to this law, but according to

$$I = Ae^{-kx}.$$

The experiments on limiting order of interference are thus inconclusive unless they are accompanied by a simultaneous investigation of the energy curve of each line. The experiments on secondary hydrogen lines have not been repeated in this way, so that the question must at present be regarded as open. At the same time, there are atomic systems present in the tubes which should have a spectrum of some type, not coinciding with the Balmer series — for instance, negatively-charged hydrogen with stationary states other than the ring. Thus the secondary spectrum must include a certain number of atomic lines and, of course, the spectrum of H_3 and its ionised molecules, but we cannot be far from the truth in ascribing the greater part of it, under ordinary circumstances, to the molecule of hydrogen—neutral for lines not showing Zeeman effect—but not of course *entirely* so of necessity, and positively charged for lines which do show this effect.

A suggestion may also be made with regard to the third type of line which is abnormal as regards the Zeeman effect, though not many of these were found by Dufour among those he examined. It is possible that they are mainly of low intensity. For it is clear that stationary states are possible in the molecule: (1) of types suitable for a Zeeman effect, *i.e.* with values of W—the energy function of Bohr—which are altered to three distinct values in the field, and (2) with values of W corresponding to types in which, for instance, nuclei play a greater part in the motion, or the electrons can neither be treated as approximately single. In a passage between two states, one of Type I. and the other of Type II., the normal Zeeman effect could not be expected to occur, and we suggest the possibility that the third type of line has a physical origin of this character.

The First Band.—We now proceed to a detailed examination of the components regarded by Fulcher as belonging to the two bands which he found. The most characteristic components are in each case arranged in triplets, and there are other components associated with them. It is most profitable to consider mainly the triplets, and we now take up those of the first band, of separation about 100 Å.

Table I.

λ.	ι.	λ.	ι.	λ.	ι.	λ.	ι.	λ.	ι.
6441·83	0	6340·75	2	6238·57	6	6135·56	6	6032·14	5
6433·80	0	6332·70	1	6230·48	2	6127·53	2	6024·00	4
6428·36	2	6327·25	8	6225·02	10	6122·01	10	6018·53	9

The separations in wave-length are:—

101·08	102·18	103·01	103·42
101·10	102·22	102·95	103·53
101·11	102·23	103·01	103·48

The wave-numbers and separations in wave-number, denoted by n and δn, in the usual notation, are:—

TABLE II.

n.	δn.	n.	δn.	n.	δn.
15523·53	247·48	15771·01	258·31	16029·32	269·11
15542·91	248·14	15791·05	259·08	16050·13	269·65
15556·07	248·59	15804·66	259·52	16064·21	270·29

n.	δn.	n.
16298·43	279·43	16577·86
16319·78	280·49	16600·27
16334·50	280·85	16615·35

It is noticeable that, for the three individual components of two triplets, the interval between corresponding components is much more nearly the same in wave-length than in wave-number. We may exhibit, for example, the numbers 101·08, 101·10, 101·11 by the side of 247·48, 248·14, and 248·59. It is therefore likely that a *simple* formula for their representation may be more easily obtained in terms of wave-length rather than wave-number—without, however, any implication of a more physical basis.

Expressed in wave-numbers, the lines do not form triplets in the sense in which the word is used in discussions of line-spectra. The frequency differences of the extreme components of each triplet become

32·54, 33·65, 34·89, 36·07, 37·49,

decreasing towards the smaller wave-number. On the other hand, there is a smaller rate of decrease in the wave-length separations which are

13·47, 13·50, 13·55, 13·55, 13·61.

The measurements taken—though far from complete in certain regions—are those of Watson, and do not agree too closely with those of Merton and Barratt. Their accuracy is not known with certainty, and cannot be so great as he claims, but these tendencies are uniform and must be regarded as real, and the lines are not triplets of constant separation, though capable of approximate treatment as such. The fact that a wave-length formula may be more suitable, in a superficial sense, for these bands leads us to inquire into the most suitable one. In the band which starts at λ6018·53, we find, as the best three-constant formula,

$$\lambda = 5966{\cdot}709 + 103{\cdot}6557(m - \tfrac{1}{2}) - 0{\cdot}052667(m - \tfrac{1}{2})^3,$$

there being no very good formula without m^3. This reproduces the lines with great accuracy below the limit of experimental error. Thus—

m.	1.	2.	3.	4.	5.
λ calc.	6018·530	6122·015	6225·025	6327·246	6428·360
λ obs.	6018·53	6122·01	6225·02	6327·25	6428·36
$\Delta\lambda$	0·00	+0·005	+0·005	−0·004	0·00

The band, in fact, extends much further into the red, several faint lines beyond H_α found in Porlezza's* list being obvious continuations.

Although Watson's† measurements for the weaker lines may be uncertain in many cases to as much as 0·06 Å., the table of comparative measurements on different plates, given in his paper, indicates that the stronger lines may be very accurately determined. This accurate representation of the present system, by a three-constant formula, supplies a further indication, for it is the strong set of the three. The other two series, fitted to a formula of any type, are found to give discrepancies as great as 0·03 Å.; and in default of better measurements, the simplest procedure for their representation is to vary only the first term of the formula above to take account of the wave-length differences, although the indications already given show that the other terms must change slightly.

With these main triplets, Fulcher associates, on experimental grounds, other sets of single lines as follows; the figure in brackets denoting intensity:—

Set (*a*) . .	6285·56(4),	6183·19(4),	6080·03(9),	5975·68(8)
Set (*b*) . .	6299·60(5),	6197·32(2),	6094·03(1),	5989·47(3)
Set (*c*) . .	6199·58(4),	6096·21(4),	5992·14(0).	
Set (*d*) . .	6303·68(0),	6201·38(2),	6098·45(3),	5994·30(4).

Their differences are all practically 100 Å. again. These sets exhibit more divergent differences among themselves, and with the other three sets, both in wave-length and in wave-number. The fact that the intensities do not run very regularly along the sets is not an unusual phenomenon in band-spectra. Certain weak lines in the red, in Porlezza's spectrum, again appear to belong to these sets, and we find it possible to extend the last set considerably. The attempt to represent set (*d*) by a four-constant formula soon shows that only three are really necessary, and in fact that the formula

$$\lambda = 6303{\cdot}656 - 102{\cdot}2319(m-1) - 0{\cdot}097845(m-1)^3$$

gives the seven observed lines (with errors below)—

$m=0$.	$m=1$.	$m=2$.	$m=3$.	$m=4$.	$m=5$.	$m=6$.
$\lambda=6405{\cdot}98(1)$	6303·65	6201·33	6098·42	5994·32	5888·46	5780·26
$\Delta\lambda=$ +0·02	−0·03	−0·05	−0·03	+0·02	+0·05	+0·02

These series also appear to be capable of extension, and among the weaker lines above λ6000 there are other significant groups of the same character, which fairly clearly belong to the same band system. We

* *Il Nuovo Cimento.*

† The corresponding treatment in terms of the measurements of Merton and Barratt is not necessary to our argument.

shall later develop the band more completely than is possible from Watson's measurements, which do not include all the weak—and sometimes fairly strong—lines since found by others. But from Watson's tables the following can be picked out and may be mentioned at this point.*

λ.	Δλ.	λ.	Δλ.	λ.	Δλ.	λ.
6045·80(0)	98·49	6144·29(0)	96·13	6240·42(0)	...	...
6044·80(0)	98·50	6143·30(0)	...	...	...	...
6041·27(1)	98·45	6139·72(0)	96·31	6236·03(0)	93·81	6329·84

The last member of the lower series was calculated from a formula, and agrees precisely with observation.

We shall see later that it is possible to regard practically all the weak lines in the region λλ6000–6400 as forming part of this band-spectrum, and in no case does a line known to show Zeeman effect come into the scheme. The band is thus very much more extensive and fundamental than Fulcher, who only found the stronger members, supposed. The general result is that all lines in this region, not showing Zeeman effect, are components of one single scattered band, and it is *à priori* probable that similar results are true for the remainder of the spectrum, so that the total number of bands required for the interpretation of the non-Zeeman spectrum is very small and not beyond the possibilities of the hydrogen molecule, in spite of its simple structure. The somewhat bewildering multitude of lines in the spectrum can thus be accounted for.

More methods are urgently needed in the laboratory of isolating special bands, as Fulcher isolated the two low-pressure bands. Merton and Barratt's Bakerian Lecture does much in this direction.

The Second Band of Fulcher.—The second band already known consists of sets of triplets mutually distant about 120 Å., and is much less conspicuous. Its members already recognised are given in the table (Watson's measurements):—

λ.	ι.	λ.	ι.	λ.	ι.	λ.	ι.
5671·10	0	5552·60	0	5435·06	1	5318·06	0
5661·97	0	5543·71	0	5426·20	1	5309·10	0
5655·98	1	5537·67	5	5420·10	4	5303·35	2

The intervals are

118·50	117·54	117·00
118·26	117·51	117·10
118·31	117·57	116·75

and the triplet separations

9·13	8·89	8·86	8·96
5·99	6·04	6·10	5·75

* Allen, *loc. cit.*, has given the band, but we consider it to be more extensive.

The fact that the band shows experimental relation to the first indicates that these numbers should run in a similar manner, though much larger errors may be expected on account of low intensity. But no possible errors can explain two outstanding discrepancies, and it is evident that the lines λλ5671·10, 5303·35 cannot be approximately correct. It seems probable that the actual band lines are weaker ones which are marked by these. The line λ5309·10 also shows a discrepancy of twice the admissible amount, after Watson, about 0·05 Å., but this is an older measurement of Hasselberg, the line, like the majority of known lines in this region, not having appeared on Watson's plates.

If, moreover, the line λ5303·35 is replaced by the line which the rest of the band involves, the stronger series shows a new member, λ5775·28, precisely in the calculated position. The best formula is

$$\lambda = 5186{\cdot}196 + 116{\cdot}7874m + 0{\cdot}04119m^3$$

involving the integer m instead of $m - \frac{1}{2}$. The average triplet separations which agree are

$$\tfrac{1}{3}(8{\cdot}89 + 8{\cdot}86 + 8{\cdot}96) = 8{\cdot}90$$
$$\tfrac{1}{3}(5{\cdot}99 + 6{\cdot}04 + 6{\cdot}10) = 6{\cdot}04$$

Using both these and the average intervals, the corrected wavelength of the 5303 line is 5303·04 (which suggests by comparison that a mere misprint has occurred in Watson's paper, for he gives 5303·35), in order to conform to the rest of the band. Three calculations on different bases give the same value. The band, in its strongest components, is exhibited in the table, the new component λ5775 being included:—

λ Calc.	λ Obs.	Δλ.
5303·024	(5303·04)	−0·016
5420·100	5420·10	0·00
5537·670	5537·67	0·00
5655·981	5655·98	+0·001
5775·281	5775·28	+0·001

As in the case of the first band, we can at once find other groups in the region λλ5300–5700 which clearly belong to the band, though Fulcher did not recognise any but the triplets in this band. Most of the lines in this region can be shown to form part of the band, but the region requires a new exploration on account of the weakness and consequent inaccuracy of most of the lines contained in it. For the majority, we have to rely mainly on Hasselberg's older measurements, which can be in error by 0·2 Å. Dufour, moreover, has not investigated the Zeeman effect at all in this region.

Relation between the two Bands.—If we represent the stronger series in the first band by a formula of the Deslandres type, in wave-number, the best which can be obtained is

$$n = 15556{\cdot}072 + 243{\cdot}313m + 5{\cdot}3767m^2,$$

which yields the errors Δn, and consequent errors $\Delta\lambda$, as follows:—

	$m=0$.	$m=1$.	$m=2$.	$m=3$.	$m=4$.
$n=$	15556·07	15804·76	16064·20	16334·40	16615·35
$\Delta n=$	0·00	+0·10	−0·01	−0·10	0·00
$\Delta\lambda=$	0·00	−0·05	0·00	+0·04	0·00

The agreement is much inferior to that obtained with the previous formula, but nevertheless there is no doubt that the Deslandres formula is at least a first approximation, in the case of bands, to a formula with a real physical basis. The present formula was obtained, as the best, from all the lines, and not on the alternate ones which happen to show no error. An average value of the third coefficient in

$$n = A + Bm + Cm^2$$

was deduced, in the first place, from the second differences, then one of B from the first differences, and finally one of A. This method was used later for the second band also.

Now the actual second differences of the observed wave-numbers are 10·96, 10·74, 10·56, showing a general gradation which, being a definite phenomenon for all strong and therefore well-measured sets of series in these bands, cannot be ascribed to errors of observation. In fact it is so regular—in cases we do not discuss in detail in this paper—with a practically constant third difference, as to prove that Watson's observations of the *strong* lines are very accurate; this is confirmed by Merton and Barratt.

It shows, moreover, that Deslandres' formula cannot apply rigorously, and a very small term in m^3 or m^4 is necessary.

It is a matter of considerable interest that these hydrogen bands deviate more from Deslandres' formula than any others for which a precise investigation has been possible. We may, in fact, state a general theorem which has become apparent to the writer, though already suggested by others, during some descriptive investigations of band-spectra which are not within the scope of this paper.

The deviations of band-components from Deslandres' formula are always of an order determined by the molecular weight of the source, and thus are naturally greatest in hydrogen. The conclusion is almost irresistible that they contain a term of order $\frac{m}{M}$, where m is the mass of an electron, and M of a nucleus in the molecule, for they certainly appear to accord with this. On the quantum theory, this implies the conclusion, based readily on other grounds, that the nuclei move during the emission of band-spectra, and probably also that the motion of these nuclei *must be quantised like that of the electrons.*

Returning to the formula for the first band (strong components), viz. with n again denoting wave-number,

$$n = 15556{\cdot}072 + 243{\cdot}313m + 5{\cdot}3767m^2$$

we can put it into the form

$$n = \alpha + \beta(m+\mu)^2$$

where μ corresponds to Rydberg's "phase" in line spectra, and α can be called the "limit," by the same analogy. The result is

$$n = 12803{\cdot}4 + 5{\cdot}3767\ (m + 22{\cdot}627)^2.$$

The "limit" on this basis is $n = 12803{\cdot}4$, corresponding to a wave-length

$$\lambda = 7810{\cdot}4,$$

which may not, of course, have physical significance in itself. Its accuracy depends very much on the mean second difference, and it is found that an error of 0·05 Å. in an individual line could alter the limiting wave-number by 20. But if the wave-lengths have the accuracy we are led to expect from the results of the investigation, the error should not be so large.

The wave-numbers for the strongest series in the second band, if the corrected value λ 5303·04 is adopted as the final line in accordance with the preceding discussion, are as follows:—

$\lambda =$	5775·28	5655·98	5537·67	5420·10	5303·04
$n =$	17315·18	17680·40	18058·14	18449·84	18857·18

The second differences are 12·52, 13·96, 15·64, again showing a regular but larger gradation. The formula calculated by the preceding method is

$$n = 17315{\cdot}188 + 357{\cdot}420m + 7020m^2$$

giving

	$m=0.$	$m=1.$	$m=2.$	$m=3.$	$m=4.$
$n =$	17315·19	17679·63	18058·11	18450·57	18857·19
$\Delta n =$	+0·01	−0·77	−0·03	+0·73	+0·01
$\Delta\lambda =$	0·00	+0·25	+0·01	−0·22	0·00

The agreement is much less satisfactory, and this cannot be due to errors of observation in the main, for the mode of calculation indicates that if Deslandres' formula were exact, all lines would be incorrect to the same average extent, every error of observation affecting every calculated line almost equally. The discrepancies are due to an incomplete formula. When thrown into the standard form, the series becomes

$$n = 12765{\cdot}7 + 7{\cdot}020(m - 25{\cdot}457)^2.$$

The "limit" is within 37 units of that of the previous band, or they are identical within the limits of error due to the insufficiency of the formula, or even due to two errors of amount 0·05 Å. in measurements. Formulæ with one more constant can be constructed readily, and make the limits even closer. Thus we reach the important conclusion that *the two Fulcher bands have the same "limit,"* in this rather special usage of the term "limit."

This is the first analytical relation to which the secondary spectrum has led.

It has always been possible—perhaps only till the advent of the

quantum theory—that the proper formula for band-spectra involves the square of the frequency in the form

$$n^2 = A \pm B(m+\mu)^2,$$

but investigation shows that this does not represent the bands with more accuracy than the Deslandres form, and the wide separation of the hydrogen bands actually gives a real test to this hypothesis, which is often made. It seems, however, that as in line-spectra, band-spectra only involve the frequency to its first power.

The individual formulæ of the Deslandres type for the three main components of the first band are

$$n_1 = 12803{\cdot}4 + 5{\cdot}3767\ (m - 26{\cdot}6270)^2$$
$$n_2 = 12810{\cdot}1 + 5{\cdot}39165(m - 26{\cdot}5135)^2$$
$$n_3 = 12767{\cdot}68 + 5{\cdot}3250\ (m - 26{\cdot}7495)^2$$

where $m = 0$ gives the triplet λλ 6018, 6024, 6032, and $m = 1, 2, \ldots$ the redder lines.

As stated before, an error of 0·02 in a single line is sufficient to give an error of 10 units in the limit, apart from errors of formula. It is therefore clear that all three limits are probably, in actual fact, identical.

The individual formulæ for the three components of the second band are

$$n_1 = 12765{\cdot}7 + 7{\cdot}0200(m - 28{\cdot}4570)^2$$
$$n_2 = 12734{\cdot}9 + 6{\cdot}940\ (m - 28{\cdot}6441)^2$$
$$n_3 = 12858{\cdot}4 + 7{\cdot}1225(m - 28{\cdot}8918)^2$$

$m = 0$ giving the triplet near λ 5303.

In the second case, Hasselberg's line was not used in the calculation, as its relation to the other measurements is uncertain. The mean limit is 12786·3, very close to 12793·7, the mean of the other three series. In view of the weakness of the second band, and the relative uncertainty of the measurements, the agreement is remarkably exact.

We shall not give the corresponding calculations, which are somewhat long, of appropriate formulæ for the sets (*a*), (*b*), (*c*), (*d*), associated by Fulcher with these bands. As we have seen, they are only four of many, the others being weaker. We have described enough detail to establish the existence of one large inter-related band, in all probability due to neutral hydrogen molecules. A more complete scheme of the band in the region 6000–6500 will be given later.

A Third Band.—Fulcher was unable to find definite experimental evidence of any band in the blue part of the spectrum, but this part is very strong in lines which show no Zeeman effect—as well as in those which do—and bands should be expected. We give some indications of a third band which can be isolated from Watson's measurements—these measurements are especially good in this region—and which can be picked out as a probable band on some plates, showing fine definition, taken some time ago by Professor Merton and the writer. The general separation is about 80 Å., and the band again includes no lines showing Zeeman effect—a point which can be tested very precisely here, for Dufour's investigation was much more complete in this region.

The following may be regarded as the main lines of this band, forming a triplet system, the separations showing a character strictly analagous to that of the two bands already investigated.

λ.	ι.	λ.	ι.	λ.	ι.	λ.	ι.
4404·80	0	...	...	4578·22	2	4661·55	3
...	...	4493·86	1	4579·60	1	4662·94	4
...	...	4498·75	1	4584·71	1	4667·90	1

λ.	ι.	λ.	ι.
4742·19	0	4819·50	0
4743·56	1	...	...

The extreme numbers in the upper row were found by interpolation to belong to the band. The intervals of the others are—

...	83·33	80·64
85·74	83·34	80·62
85·96	83·19	...

Another portion of the same band is

λ.	λ.	λ.	λ.	λ.
...	...	4507·05	4590·06	...
...	4420·48	4505·80	4588·85	4669·39
4329·84	...	4502·11	4585·21	4665·73

the separations being remarkably constant, and the extreme members being calculated. The intervals are practically as before. The doublet members

4426·11(1)	4511·89(2)	4675·41(1)
4425·33(1)	4511·11(2)	4674·62(1)

of separations 0·78, 0·78, 0·79 may also be mentioned. The general character of this band exactly corresponds with that of the others, and perhaps these twenty-eight members are a sufficient indication of its existence, though many other sets of single lines apparently belong to it, and the complete band appears to account for the greater number of lines in the blue, which do not show a Zeeman effect.

This band can be represented by Deslandres' formula with a degree of closeness exactly comparable with Fulcher's second band. For example, the long series of lines in the first table,

$$\lambda\lambda 4404{\cdot}80,\quad 4578{\cdot}22,\quad 4661{\cdot}55,\quad 4742{\cdot}19,\quad 4819{\cdot}50,$$

—where the last measurement is due to Porlezza, the line, which is weak, not being on Watson's plates—can be represented by the formula

$$n = 20749{\cdot}22 + 324{\cdot}934m + 13{\cdot}152m^2,\ (m = 0,\ 1,\ 2,\ \ldots)$$

which gives errors of magnitude

$\Delta n =$	$+0{\cdot}18,$	$+0{\cdot}01,$	$-0{\cdot}37,$	$0{\cdot}00,$	$+0{\cdot}18$
$\Delta\lambda =$	$-0{\cdot}03,$	$0{\cdot}00,$	$-0{\cdot}06,$	$0{\cdot}00,$	$-0{\cdot}03$

Its limit, calculated from the four strongest lines, is

$$n = 18714{\cdot}9$$

The components next in order of intensity are

$$\lambda\lambda 4493{\cdot}86,\quad 4579{\cdot}60,\quad 4662{\cdot}94,\quad 4743{\cdot}56$$

and admit the formula

$$n = 21081{\cdot}22 + 351{\cdot}3475m + 13{\cdot}0375m^2$$

with errors

$$\Delta\lambda = 0{\cdot}00,\quad -0{\cdot}02,\quad +0{\cdot}02,\quad 0{\cdot}00$$

and the limit calculated from this formula is 18714·0, within one unit of that of the other components. It is not that of Fulcher's bands, and, in fact, the writer does not consider that this band is due to the same source as Fulcher's but is one of a similar character given by a similar, but different, molecular arrangement whose origin is not discerned in this paper. If this band also belonged to the Fulcher system, it would moreover have appeared in his experimental investigation.

All the sets in the Fulcher bands do not possess the same limit, although the fundamental triplet sets do. The same phenomenon occurs here also. The portions of this band already discussed have the same limit, but other limits also exist. For example, the sets

	$\lambda\lambda 4420{\cdot}48,$	4505·80,	4588·85,	4669·39
$\lambda\lambda 4329{\cdot}84,$		4502·11,	4585·21,	4665·73

tend to the common limit 18912·8 with great accuracy. The limits in this band are much more closely determinable than in Fulcher's for the measurements are more reliable, and other measurements by Ames are available in this region for purposes of comparison.

Other Bands.—That similar regularities of the same band-spectrum type exist throughout the spectrum is quite clear. It is, moreover, fairly certain that the whole spectrum not showing Zeeman effect, in spite of its multiplicity, is of this character. Further evidence as to the great ramifications of one individual and typical band will appear later.

We may conclude with an illustration of yet another remarkable band in a new region of the spectrum which is densely crowded with lines. We may cite a remarkable group of lines about λ3870, for out of 17 successive lines in the table of wave-lengths, 10 may be arranged as doublets in the following way:—

3861·69 / 3863·35 } 1·66	3865·62 / 3867·29 } 1·67	3869·33 / 3870·98 } 1·65
3870·09 / 3871·75 } 1·66	3878·08 / 3879·73 } 1·65	

These evidently indicate another ordered arrangement with very small band-separations, and the remainder of this band can be followed in some detail.

But the further citation of individual bands of this nature in detail would serve no useful purpose at the moment. The complete unravelling of the spectrum can in fact only be performed by the help of plates of spectra, which have been mainly instrumental, rather than tables of wave-lengths, in enabling the present writer to arrive at the conclusions in this section. Our purpose has been merely to show that the secondary spectrum of hydrogen, in so far as it shows no Zeeman effect, is entirely a scattered band-spectrum, and to determine some of the mathematical structure of an individual band with a view to a theory of its origin which we develop afterwards. The general similarity of the bands shows that this structure is of the same type in all cases.

We shall discuss the lines showing Zeeman effect, which are of a very different type, later.

General Conclusions.

1. The two Fulcher bands are numerically related, and form a single system.

2. The lines of the secondary spectrum, not showing Zeeman effect, are of the form of widely separated bands analogous to those of Fulcher, and it is probable that the number of distinct bands is very small.

Note on Spectroheliograms taken with Different Parts of the H_α Line.

By T. Royds, D.Sc. (Plate 13.)

When the second slit of a spectroheliograph is sufficiently narrow and is set centrally on the H_α line, the resulting spectroheliogram presents well-known features of which the most noteworthy are (1) dark markings, and (2) bright flocculi, whilst the general surface of the sun is represented by a fine network or réseau in which the photographic contrast is not large. This réseau is due to the presence of innumerable faint reversals of the H_α line rather than to darkenings of the H_α line.

The routine H_α spectroheliograms of the Kodaikanal Observatory are required principally for the record of the dark markings. Since the darkenings of the H_α line causing dark markings generally extend across the whole width of the line, the record of dark markings is preserved when the second slit of the spectroheliograph has a considerable width, say 0·8 Å.* It is found that with a slit width of 0·4 Å. the dark markings are practically identical, whether the middle of the slit or either of its edges is central on the H_α line. Consequently, if the slit is widened to about 0·8 Å., the dark markings are recorded with a shorter exposure, but of course at the expense of a smaller resolving power, which is unimportant for these features.

The second slit of the spectroheliograph cannot be made much wider than 0·8 Å. without encroaching on the edges of the H_α line, and as soon

* Slit widths of this order of magnitude have been used by Mr. J Evershed, F.R.S., for H_α spectroheliograms at Kodaikanal.

as this happens the characteristics of the resulting spectroheliogram begin to change completely. The irregular edges are perhaps the most obvious feature of the H_α line in the sun, and as soon as the slit is wide enough to include the edges of the H_α line, or even is kept narrow but displaced far enough from the centre to include either edge of the H_α line, continuous spectrum passes through the slit in an amount corresponding to the irregularity of the edge of the H_α line. In fact, the character of the H_α spectroheliogram is now principally determined by the varying amounts of continuous spectrum, and darkenings and reversals of the H_α line play a minor part.

If the second slit of the spectroheliograph has a width of about 0·5 Å. and is moved from the centre of the H_α line step by step towards either edge of the line, no remarkable change in the resulting spectroheliogram takes place until the centre of the second slit is about 0·3 Å. from the centre of the H_α line. Thereafter continuous spectrum begins to affect the spectroheliogram. Dark markings and bright flocculi become less conspicuous as the slit is moved further away from the centre of the H_α line, and by the time the mean edge of the H_α line is about the centre of the slit they are generally not to be traced. Very striking is the much coarser network or réseau over the general surface of the sun, and the contrast in the réseau is much greater than in the spectroheliogram with central setting.

The H_α line is often shifted by Doppler displacements (sometimes in both directions simultaneously), and these play a part in the interpretation of spectroheliograms when the slit is set on the edge of the H_α line. These Doppler effects are not under discussion in this paper, but only the effects of the varying width of the H_α line, *i.e.* expansion or contraction of both edges of the H_α line simultaneously, such as is generally interpreted as due to variations in density or pressure or both in the lower regions of the sun's atmosphere.

Two striking features and one less striking are revealed in the vicinity of sunspots in spectroheliograms obtained when the slit of about 0·5 Å. width is set on one edge of the H_α line. (1) A bright spot ring, such as that illustrated in fig. 1, 1924 July 3, is to be seen, often complete and unbroken, round nearly all spots of medium size. In such spots the H_α line is usually narrower than normal just outside the penumbra for a distance approximately equal to the radius of the spot. When the slit is set on the edge of the H_α line, more continuous spectrum than normal falls on the photographic plate where the line is narrow, thus causing in the spectroheliogram a bright ring round the sunspot. The narrowing of the H_α line round spots is often accompanied by a reversal of the line, so that the spectroheliogram obtained with the slit central on the H_α line will also often show a bright ring round the spot; but such rings are never so conspicuous as when the slit is on the edge of the line, for in the latter the reversal combines with the narrowing of the line to enhance the brightness round the sunspot. Bright rings round spots in spectroheliograms with the slit set in the middle on the line were first noticed by Hale, using the H_δ line.*

* Hale, *Astroph. Journ.*, **23**, 54, 1906.

(2) Outside the bright ring round a sunspot the H_α line is wider than normal, and there it produces a dark area when the slit is set on the edge of the line. These dark areas are the well-known "dark hydrogen flocculi." * They are, however, quite distinct from "dark markings." Dark markings are best seen in spectroheliograms obtained with the slit set in the middle of the H_α line, whereas the dark flocculi cannot be shown with the H_α line unless the edge of the line is on the slit. Fig. 2 of 1920 October 15 shows an extensive dark flocculus round a sunspot group when the slit was set on the edge of the H_α line. Fig. 3 shows the same region as fig. 2, but when the edge of the H_α line was excluded from the slit of the spectroheliograph, *i.e.* when the slit was central on the H_α line. Spectroheliograms obtained with the slit central on the H_α line generally show a delicate tracery of fine "dark markings" and of bright flocculi round sunspots, but those obtained with the slit on the edge of the H_α line, as in fig. 2, show a dark flocculus comparatively massive and featureless.

(3) A less striking feature in spectroheliograms obtained with the slit on the edge of the H_α line is a bright surround which intervenes between the dark flocculus and the coarse réseau of the general surface of the sun. In fig. 2 the réseau of the general surface can be seen near the edge of the figure, but it will be seen that the dark flocculus is bounded by a region in which the bright patches are considerably more extensive than those in the general undisturbed surface of the sun. The bright patches in the surround of the dark flocculus are often interrupted by dark features combining to suggest the spot vortex, which is generally best exhibited in the delicate tracery obtained only when the slit is central on the H_α line.

Either the red or the violet edge of the H_α line gives the above three features, generally without any essential differences.

It is believed that these three features are typical of the majority of sunspot disturbances, and that the story they tell of what is taking place in the lower levels of the sun's atmosphere is an essential part of a spot disturbance.

Explanation of the Plate.

Fig. 1. 1924 July 3, showing a bright ring round the spot, with diameter about twice that of the spot. The slit of the spectroheliograph was set on the violet edge of the H_α line, but not so far from the centre as to obliterate the dark markings. The dark flocculus and bright surround are not well developed in consequence.

Fig. 2. 1920 October 15, showing dark flocculus and its bright surround. The slit of the spectroheliograph was set on the red edge of the H_α line. The spot ring extends only halfway round the spot.

Fig. 3. 1920 October 15, but with the slit set nearly centrally on the H_α line. There is here no suggestion of the features in fig. 2.

The diameter of the sun is approximately three times the width of figs. 2 and 3.

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* Hale and Ellerman, *Proc. Roy. Soc.*, A, **83**, 177, 1909.

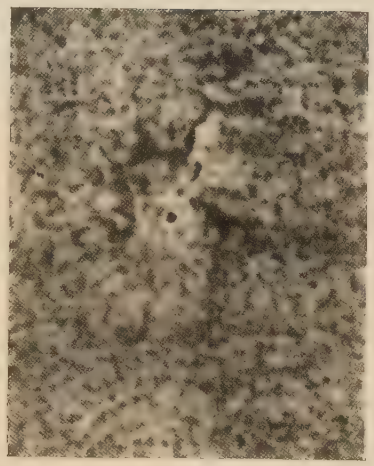


Fig. 1. 1924 July 3.

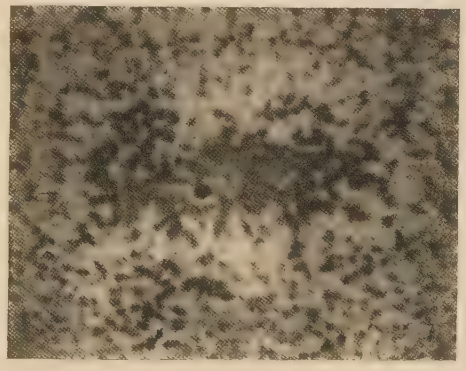


Fig. 2. 1920 Oct. 15.

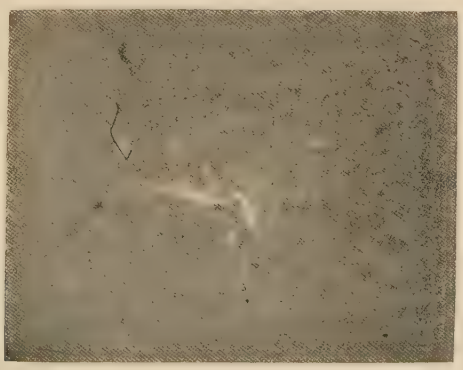


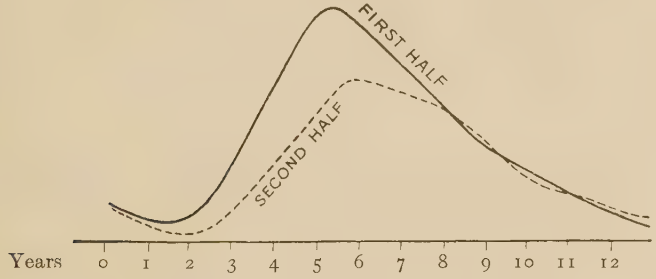
Fig. 3. 1920 Oct. 15.

Note on the Alternation of the Eleven-year Solar Cycles.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The discovery that the magnetic polarity of sunspots is reversed at minimum, and then maintained until the next minimum, calls attention to an essential difference between one solar cycle and the next. A difference of this kind was noted in 1913 (*M.N.*, **74**, 94) before the magnetic reversal at minimum was discovered. It is there stated:

> Now in the first place we find that there is not a satisfactorily uniform period of 11½ years, but rather a double period of 23 years, the halves of which are not quite symmetrical.



Alternate Halves of 23-Year Cycle compared.

The diagram given from an analysis of Wolf's numbers may perhaps be reproduced here for convenience.

2. More material is now available, and it seems desirable to re-examine the point. But instead of taking Wolf's numbers we may now take the Greenwich measures, which have been published up to 1923 (see *M.N.*, **84**, 742; **75**, 19; **63**, 466; and **49**, 382). Both the N and S mean latitudes have been subjected to harmonic analysis; as also the logarithms of the N and S spotted areas (whole spots). The logarithms were used rather than the areas themselves to avoid the swamping influence of the large numbers at maximum. Harmonic analysis is specially simple when 12 terms are used; and though 12 years is longer than the cycle, the analysis in the paper cited indicated that it was not much longer than the present value of the cycle, the length of which will, of course, appear from the progressive change of maximum. Hence 4 sets of 12 years ending with 1923, as shown in Table I., were analysed.

Table I.

1876–1923. *N Latitudes.* *Unit* +0°·1.

126	91	71	236	200	182	160	110	107	105	99	88
71	73	222	205	151	149	123	143	136	83	98	62
66	86	188	181	163	117	140	101	104	95	86	79
205	248	211	180	151	138	120	98	113	73	88	139

1876–1923. *S Latitudes. Unit* −0°·1.

109	95	81	225	194	183	193	139	117	124	105	83
75	119	218	199	217	143	156	125	148	77	108	104
83	163	153	212	169	156	140	138	103	99	112	60
78	201	231	198	170	157	137	118	100	89	66	173

1876–1923. *N log (Areas). Unit* 0·01 *in Logarithm.*

151	142	138	115	241	270	265	253	268	245	188	164
130	70	172	260	278	271	273	275	231	229	204	136
141	134	162	212	243	288	273	269	250	248	182	123
0	70	200	258	267	293	278	275	232	241	221	152

1876–1923. *S log (Areas). Unit* 0·01 *in Logarithm.*

195	181	0	154	220	236	275	291	278	272	248	213
184	186	167	223	278	297	287	261	253	250	242	194
169	82	132	232	234	264	238	277	258	259	230	167
156	40	172	250	240	283	271	269	261	220	196	134

3. Submitting these to harmonic analysis, we get the results of Table II. The third and fourth harmonics are small, and have been omitted.

TABLE II.

N Latitudes.

(1) 1876–87 $13^{\circ}\cdot1+5^{\circ}\cdot0\cos(\theta-129^{\circ})+2^{\circ}\cdot9\cos 2(\theta-125^{\circ})$
(2) 1888–99 $12\cdot6+5\cdot1\cos(\theta-125)+3\cdot5\cos 2(\theta-76)$
(3) 1900–11 $11\cdot7+4\cdot4\cos(\theta-116)+2\cdot6\cos 2(\theta-85)$
(4) 1912–23 $14\cdot7+6\cdot6\cos(\theta-54)+2\cdot8\cos 2(\theta-26)$

S Latitudes (signs reversed).

(1) 1876–87 $13^{\circ}\cdot7+5^{\circ}\cdot4\cos(\theta-147^{\circ})+2^{\circ}\cdot1\cos 2(\theta-125^{\circ})$
(2) 1888–99 $14\cdot1+5\cdot4\cos(\theta-116)+2\cdot5\cos 2(\theta-84)$
(3) 1900–11 $13\cdot2+4\cdot9\cos(\theta-115)+1\cdot9\cos 2(\theta-76)$
(4) 1912–23 $14\cdot3+5\cdot8\cos(\theta-83)+1\cdot3\cos 2(\theta-57)$

N log (Areas).

(1) 1876–87 $2\cdot03+0\cdot74\cos(\theta-207^{\circ})+0\cdot13\cos 2(\theta-158^{\circ})$
(2) 1888–99 $2\cdot11+0\cdot84\cos(\theta-180)+0\cdot33\cos 2(\theta-110)$
(3) 1900–11 $2\cdot10+0\cdot78\cos(\theta-184)+0\cdot10\cos 2(\theta-98)$
(4) 1912–23 $2\cdot07+1\cdot03\cos(\theta-180)+0\cdot53\cos 2(\theta-102)$

S log (Areas).

(1) 1876–87 $2\cdot14+0\cdot90\cos(\theta-233^{\circ})+0\cdot31\cos 2(\theta-157^{\circ})$
(2) 1888–99 $2\cdot35+0\cdot55\cos(\theta-190)+0\cdot21\cos 2(\theta-135)$
(3) 1900–11 $2\cdot12+0\cdot73\cos(\theta-205)+0\cdot29\cos 2(\theta-114)$
(4) 1912–23 $2\cdot08+0\cdot85\cos(\theta-186)+0\cdot26\cos 2(\theta-110)$

4. *Constant Terms.*—The mean N latitude of spots is 1°·25 greater in the even cycles (2) and (4) than in the odd and the mean S latitude 0°·75 greater. We may say that the spots are 1° further from the equator in the even cycles. The mean log (areas) are also slightly greater in the even cycles, +·025 for N and +·085 for S : mean of both +·055. The mean spotted areas themselves would be about as 126 and 143.

5. *Differences of Phase.*—We can assess the difference of phase between odd and even cycles before assigning the period, if we may assume that the period, whatever it may be, is uniform. For each set gives us two measures, excess of even cycles over odd cycles, viz. $2-(1+3)/2$ and $(2+4)/2-3$. These are as follows :

Table III.

	$2-(1+3)/2$.		$(2+4)/2-3$.		Mean.
	1st Harm.	2nd Harm.	1st Harm.	2nd Harm.	
N Lat.	+ 2°·5	−29°·0	−26°·5	−34°·0	−21°·8
S Lat.	−15·0	−16·5	−15·5	− 5·5	−13·1
N log (area)	−15·5	−18·0	− 4·0	+ 8·0	− 7·4
S log (area)	−29·0	− 0·5	−17·0	+ 8·5	− 9·5
	−14·2	−16·0	−15·8	− 5·8	−13·0

There is thus a difference of about 13° in phase between two contiguous cycles. It is shown larger by latitudes than by areas, and larger by the first harmonic than by the second : but at present we may be content with the mean value 13°·0 for all four sets and for both harmonics. (Since a year is 30°, 13°·0 is 5·2 months.)

6. *Period.*—Adding then 13° to the even cycles, the N latitudes give us the consecutive phases :

1st harmonic . .	129°	138°	116°	67°
2nd harmonic . .	125	89	85	39

Both show a decrease. We may give the difference between extremes 3 times the weight of that between the intermediates ; and the decrease per period is then found to be 20°·8 from the first harmonic and 26°·2 from the second. Doing the same for the other sets we get :

Table IV.

Decrease of Phase per Period.

	1st Harm.	2nd Harm.		1st Harm.	2nd Harm.
N Lats.	20°·8	26°·2	N log (areas)	5°·1	15°·4
S Lats.	17·7	18·6	S log (areas)	10·0	13·6
Mean	19·2	22·4	Mean	7·6	14·5

The decrease shown by the log (areas) is systematically smaller than that shown by the latitudes. The mean for latitudes is 20°·8, and for log

(areas) only 11°·0. The period corresponding to the former is 11·35 years, and to the latter 11·65 years. The difference of 0·30 year would accumulate during the period under review to more than a year. Suppose we adopt the mean decrease of 16° per cycle, and correct all the phases for this and the even-cycle defect, reducing all four to No. (3) (which begins in 1900) as standard. Then the mean values come out :

TABLE V.

Mean Values.

	1st Harm.	2nd Harm.		1st Harm.	2nd Harm.
	°	°		°	°
N Lats.	104·5	76·5	N log (areas)	186·2	115·5
S Lats.	113·8	84·0	S log (areas)	202·0	127·5
Adopt. mean	110	80		194	120

And using the adopted mean values for both N and S the residuals for the successive cycles are :

TABLE VI.

Phase Residuals.

N Lats.		S Lats.		N log Areas.		S log Areas.		Mean.
1st H.	2nd H.	1st H.	2nd H.	1st H.	2nd H.	1st H.	2nd H.	
°	°	°	°	°	°	°	°	°
−13	+13	+5	+13	−19	+ 6	+ 7	+ 5	+2
+12	− 7	+3	+ 1	−17	−13	− 7	+12	−2
+ 4	+ 5	+5	− 4	−10	−22	+11	− 6	−2
−27	−25	+2	+ 6	+15	+11	+21	+19	+3

We see that the discrepancy between the latitudes and the log areas is chiefly due to the abnormal behaviour of the N latitudes, which indicate a shorter period than that adopted, while the evidence of the areas both N and S is in favour of a longer period. The contrast is chiefly in the last cycle.

7. *Coefficients.*—The coefficients were left until after the phases had been discussed, since if their deviations had been large it would have been necessary to multiply the coefficients by the cosines of these deviations. But the residuals in Table VI. are so small that we can use the coefficients practically as they stand, and we see that the values of even minus odd are :

N Lats.		S Lats.		N log Areas.		S log Areas.	
1st H.	2nd H.	1st H.	2nd H.	1st H.	2nd H.	1st H.	2nd H.
°	°	°	°				
+1·15	+0·40	+0·45	−0·10	+0·18	+0·32	−0·12	−0·06

The differences are not very striking, and we may say that the chief differences are in phase and mean value. But since the coefficients on the whole increase in the same sense as the mean values, the excess of the even cycles will be most conspicuous at maximum.

Summary.

It was remarked in 1913 that there was a notable difference between alternate sunspot cycles of $11\frac{1}{2}$ years, pointing to a major cycle of 23 years. The recent discovery of magnetic reversal puts this difference on a new plane.

To investigate it further from the statistical standpoint, the Greenwich measures 1876–1923 have been analysed harmonically in 4 sets of 12 years each: for N and S latitudes separately and for N and S areas (logarithms taken).

The "even" cycles 1888–99 and 1912–23 differ from the "odd" cycles 1876–87 and 1900–11 as follows:—

(*a*) The spots are in the mean 1° further from the equator in both hemispheres, and the spotted area larger in the ratio of 143 to 126.

(*b*) The phases of the variation are earlier by about 5·2 months, both as regards latitude and area: the ranges are also slightly greater.

(*c*) The latitudes indicate a mean period of 11·35 years and the areas a mean period of 11·65 years, the mean being closely 11·50 years, for the 48 years analysed.

The Stellar Magnitude Scales of the Astrographic Catalogue. Nineteenth Note: The Oxford Magnitudes for +31° to +25°. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The Oxford counts for zone +28° were given in some detail in one of the early papers of this series, before the method of clearing from the galactic condensation was adopted. Counts for the remaining zones are given below, and the whole seven reduced by the standard process adopted in 1917 November (*M.N.*, **78**, 55). So many zones have now been counted and reduced in this way that it is possible to forecast approximately the nature of a complete reduction, which will be attempted as soon as the remaining counts already made have been given.

2. The Oxford counts were made for every unit of *d*, the diameter of the photographic image; and in this complete form they reveal various features of decimal equation, etc., which it is intended to discuss in a separate publication of the University Observatory. Here the main purpose is a general comparison with other zones, and the selected points for which d=44, 34, 25, 17, 11, and for "All" stars on the plates will suffice.

[Table

TABLE I.

N for Zone +31°, Oxford.

R.A. h	44.	34.	25.	17.	11.	All.
0	13	38	94	399	1087	1653
1	17	54	137	462	1197	1790
2	16	62	155	445	1329	1973
3	23	60	140	374	832	1308
4	10	33	101	287	790	1244
5	43	155	428	1486	4222	5277
6	45	129	354	1017	2163	2986
7	29	93	248	765	1890	2505
8	10	39	149	486	982	1367
9	27	69	142	412	752	1137
10	20	59	135	359	861	1290
11	13	35	111	274	701	1211
12	20	56	154	379	875	1294
13	14	51	131	382	808	1317
14	12	48	151	380	989	1726
15	34	77	198	480	1168	1976
16	15	56	145	429	1058	1915
17	33	102	252	729	1689	2940
18	34	108	300	798	1795	5776
19	77	199	586	1967	4997	11004
20	62	208	546	1954	4742	7613
21	22	100	312	938	2509	4367
22	21	94	221	780	2009	3414
23	25	70	187	616	1347	2249

TABLE II.

N for Zone +30°, Oxford.

44.	34.	25.	17.	11.	All.
18	50	109	309	658	1224
19	43	129	441	1161	1624
20	66	138	385	1108	1544
13	43	129	308	764	1269
18	42	113	321	795	1618
51	139	387	1002	2503	3975
31	96	301	892	2402	4018
14	66	202	666	1746	2930
11	45	128	384	1036	1453
17	63	187	426	990	1526
19	65	156	418	832	1670
24	68	116	357	620	997
24	57	135	374	690	1321
11	46	117	406	1033	1713
19	63	157	445	885	1499
22	86	201	606	1128	1940
32	87	256	742	1814	3449
40	137	370	1258	4660	5935
48	152	376	1317	2484	5439
147	425	1039	3175	6723	15355
112	374	1132	2953	4688	11970
55	148	402	1150	2274	5802
25	74	236	687	1394	3227
14	32	95	230	582	2249

TABLE III.

N for Zone +29°, Oxford.

R.A. h	44.	34.	25.	17.	11.	All.
0	26	59	148	363	778	1846
1	29	74	182	528	1146	2086
2	31	83	166	546	1413	2457
3	31	85	184	536	1566	2815
4	18	46	111	233	500	1467
5	35	93	169	438	948	2863
6	30	84	201	569	1464	4158
7	15	38	96	399	1027	2984
8	21	69	172	547	1595	2637
9	15	52	142	437	831	2049
10	11	48	128	360	668	1689
11	19	50	143	336	620	1037
12	20	39	86	247	664	1191
13	8	27	69	183	597	1070
14	12	29	59	198	540	1534
15	19	46	93	315	698	1389
16	14	43	106	297	745	1602
17	21	55	165	458	910	2941
18	26	84	206	603	1316	5140
19	71	257	691	2068	4025	11683
20	25	70	216	819	2196	4768
21	25	70	185	470	1174	3133
22	15	40	120	379	807	2406
23	14	87	123	416	1011	2817

TABLE IV.

N for Zone +27°, Oxford.

44.	34.	25.	17.	11.	All.
52	86	163	401	793	1625
27	64	176	490	1051	2443
21	66	176	391	726	1604
21	42	119	381	866	2091
14	30	53	210	419	967
48	108	265	656	1361	2925
20	61	155	465	1181	2783
36	76	229	836	1919	3914
33	92	173	425	915	2174
9	28	83	276	675	1680
6	25	75	201	430	1301
15	40	114	461	860	1399
33	69	128	376	647	1268
12	34	84	195	386	921
15	39	76	236	471	1051
13	28	91	420	754	1234
13	31	80	206	402	1116
14	49	161	448	1115	3484
23	63	174	586	1599	5895
49	152	413	1161	2586	7712
41	118	281	979	2168	5992
17	52	172	590	1345	4050
12	39	88	292	761	2907
28	57	118	322	642	1875

Figures for zone +28° have been given in *M.N.*, **75**, 466.

TABLE V.

N for Zone +26°, Oxford.

R.A. h	44.	34.	25.	17.	11.	All.
0	16	36	105	328	804	2188
1	4	21	67	222	646	1883
2	15	49	112	349	793	2026
3	22	71	164	459	915	1875
4	19	42	111	267	721	1339
5	37	95	274	758	1573	4576
6	22	89	306	904	2002	5146
7	24	65	208	550	1215	2843
8	20	54	132	447	917	1979
9	11	38	110	287	691	1584
10	12	39	103	304	706	1722
11	7	28	69	288	651	1133
12	35	73	164	373	809	1382
13	12	37	93	253	587	1158
14	25	64	153	434	821	1570
15	15	35	93	327	716	1390
16	11	28	93	356	815	2294
17	24	79	223	708	1948	6445
18	42	97	289	846	2523	6382
19	68	180	446	1276	3383	6575
20	73	202	495	1566	4197	9118
21	24	85	230	697	1843	4572
22	14	51	129	452	1181	2841
23	19	58	132	372	831	1743

TABLE VI.

N for Zone +25°, Oxford.

44.	34.	25.	17.	11.	All.
8	47	112	455	1546	3187
28	68	155	477	1343	2573
29	62	156	388	837	1811
56	109	274	651	1420	2537
32	59	113	268	552	1469
50	151	531	1504	3184	5947
45	145	568	1618	3868	6946
48	112	285	903	1994	4400
26	85	298	790	1578	2753
26	60	185	445	908	1676
20	67	175	484	894	1754
11	45	105	298	544	1089
26	47	130	333	681	1091
41	127	301	861	1339	2401
22	55	142	399	723	1372
21	61	139	400	772	1669
10	25	89	382	856	1830
43	99	299	969	1938	4022
27	92	242	1062	2703	5759
45	140	383	1255	3290	5841
31	87	281	1148	3018	5513
31	85	261	942	2533	4371
19	66	178	840	2726	3914
19	45	136	441	1173	2477

3. The quantity N tabulated is the cumulative count for all stars brighter than the limit indicated in the heading: 44 denoting an image with diameter ·044 of a réseau interval (or 13″·2), 34 a diameter ·034 (or 10″·2), and so on. Three formulæ have been adopted during the Oxford work for translating measured diameters into star magnitudes. as below.

Formula.	44.	34.	25.	17.	11.	All.
$15{\cdot}0-1{\cdot}02\sqrt{d}$ . .	8·2	9·0	9·9	10·8	11·6	(13·0)
$16{\cdot}0-1{\cdot}25\sqrt{d}$ . .	7·8	8·7	9·8	10·8	11·9	(13·0)
$16{\cdot}5-1{\cdot}36\sqrt{d}$ . .	7·5	8·6	9·7	10·9	12·0	(13·0)

Moreover, the constants (15·0, 16·0, 16·5) are subject to variation with the particular plate: so that counts to a definite unit of diameter only represent roughly the same magnitude. But the number of plates is so large that the mean results are probably free from gross error, and sufficiently good for a first approximation. In what follows the magnitudes as such are not further considered, the groups being

characterised by the mean log N_0, where N_0 represents N corrected to represent a count on the galactic equator.

4. These numbers were then treated by the process described in *M.N.*, **78**, 55 (and used in subsequent papers), which clears them of the adopted galactic condensation, and the results analysed harmonically. It will perhaps be sufficient to give the results of this analysis.

TABLE VII.

Harmonic Analysis for +31°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·16	−·109	−·072	−·093	−·057	+·047	+·001	+·003	+·012
34	0·71	−·123	−·080	−·087	−·028	+·001	+·022	+·028	−·023
25	1·18	−·127	−·114	−·082	−·015	−·028	+·016	+·058	−·013
17	1·69	−·099	−·082	−·106	+·008	−·016	+·046	+·080	−·012
11	2·11	−·131	−·076	−·081	+·035	−·048	+·038	+·085	−·017
All	2·33	−·221	−·079	−·078	+·027	−·024	+·028	+·083	−·003

TABLE VIII.

Harmonic Analysis for +30°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·22	−·238	−·023	−·098	−·037	+·018	−·098	+·033	+·048
34	0·76	−·243	−·112	−·115	−·045	+·022	−·061	+·060	+·002
25	1·22	−·231	−·109	−·125	−·069	+·004	−·039	+·053	−·027
17	1·72	−·260	−·145	−·106	−·063	+·008	−·030	+·081	−·026
11	2·10	−·222	−·142	−·072	−·087	+·034	+·018	+·103	−·024
All	2·39	−·273	−·097	−·109	−·037	−·003	+·009	+·068	−·008

TABLE IX.

Harmonic Analysis for +29°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·12	+·002	+·034	+·012	+·020	+·073	−·024	+·035	+·012
34	0·62	−·105	+·017	−·021	+·056	+·102	+·011	+·029	+·003
25	1·04	−·038	−·018	−·062	+·062	+·116	−·021	+·022	−·023
17	1·55	−·052	−·057	−·076	+·082	+·125	+·001	+·062	−·045
11	1·94	−·048	−·075	−·040	+·088	+·088	+·023	+·101	−·076
All	2·36	−·080	−·062	−·080	+·046	+·068	+·037	+·074	−·033

TABLE X.

Harmonic Analysis for +28°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·11	−·007	+·002	+·065	+·056	+·047	−·069	+·088	+·071
34	0·59	−·073	−·061	+·027	+·019	+·078	−·052	+·063	+·033
25	1·03	−·099	−·094	·000	+·043	+·091	−·024	+·054	+·034
17	1·53	−·157	−·172	−·057	+·069	+·091	+·003	+·035	+·049
11	1·90	−·142	−·193	−·117	+·085	+·031	+·017	+·019	+·047
All	2·38	−·097	−·156	−·140	+·068	−·003	+·024	+·038	+·040

TABLE XI.

Harmonic Analysis for +27°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·07	+·021	+·043	−·004	+·076	−·013	+·065	+·186	+·103
34	0·52	−·005	−·003	−·053	+·085	+·014	+·037	+·184	+·067
25	0·96	−·035	−·035	−·063	+·073	+·059	+·041	+·157	+·047
17	1·48	−·059	−·082	−·062	+·083	+·048	+·014	+·108	+·038
11	1·83	−·054	−·079	−·093	+·074	+·049	+·014	+·100	+·029
All	2·26	−·118	−·045	−·138	+·057	+·072	+·013	+·075	+·020

TABLE XII.

Harmonic Analysis for +26°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·04	−·119	−·049	−·096	−·059	−·056	−·050	+·075	+·045
34	0·55	−·094	−·049	−·098	−·005	−·039	−·066	+·072	+·032
25	1·02	−·087	−·071	−·088	−·017	−·032	−·029	+·058	+·038
17	1·54	−·122	−·092	−·077	+·010	+·023	−·009	+·053	+·035
11	1·94	−·152	−·078	−·083	+·023	+·005	−·022	+·046	+·038
All	2·33	−·157	−·054	−·083	+·004	+·037	+·026	+·042	+·051

TABLE XIII.

Harmonic Analysis for +25°.

d.	log N_0.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
44	0·20	+·072	−·081	+·034	−·035	+·004	−·044	+·060	−·079
34	0·68	+·036	−·098	−·007	+·009	−·028	+·003	+·070	−·031
25	1·16	+·035	−·124	−·044	−·003	−·051	+·030	+·070	−·007
17	1·69	−·052	−·103	−·069	+·011	−·064	+·067	+·069	+·009
11	2·07	−·069	−·022	−·091	+·039	−·065	+·084	+·057	+·026
All	2·39	−·073	−·038	−·052	+·041	−·043	+·101	+·065	+·024

5. In the discussion of individual zones it has been the custom to determine the rate of change of these harmonics for 1·00 in log N_0 at the foot of each table corresponding to Tables VII. to XIII. In the present instance it seems better to collect together these rates of change in one table. They are deduced in each case by subtracting the sum of the rows for $d=34$ and $d=25$ from the sum of those for $d=11$ and All, and then dividing by the difference of log N_0.

TABLE XIV.

Rates of Change of the Harmonics for 1·00 in log N_0.

Zone.	sin θ.	cos θ.	sin 2θ.	cos 2θ.	sin 3θ.	cos 3θ.	sin 4θ.	cos 4θ.
+31°	−·040	+·015	+·004	+·041	−·018	+·011	+·032	+·006
+30	−·008	−·007	+·023	−·004	+·002	+·051	+·023	−·003
+29	+·006	−·052	−·014	+·006	−·027	+·027	+·047	−·034
+28	−·025	−·073	−·107	+·034	−·053	+·044	−·023	+·008
+27	−·051	−·033	−·044	−·010	−·019	−·020	−·064	−·025
+26	−·047	−·004	+·007	+·018	+·042	+·037	−·016	+·007
+25	−·081	+·062	−·035	+·028	−·011	+·058	−·007	+·034
Mean	−·035	−·013	−·024	+·016	−·007	+·030	−·001	−·001

6. Attention has hitherto been paid chiefly to the phases for the first harmonic, the mean value of which may be expressed in the form

$$-\cdot035 \sin\theta - \cdot013 \cos\theta = -\cdot037 \cos(\theta - 70°),$$

the coefficient being given the negative sign to indicate an obscuration. It was suggested in *M.N.*, **76**, 157 (from preliminary counts not cleared from galactic condensation) that the maximum obscuration followed the spiral

$$\alpha = 247° - 3\cdot66\delta = 146° \text{ for } \delta = 28°.$$

The value 70° is not in good accordance with 146°. For the separate zones the corresponding phases are

Zone.	Obsd.	Spiral.	O–C.
°	°	°	°
+31	110	133	− 23
+30	48	137	− 89
+29	353	140	−149
+28	19	144	−125
+27	57	148	− 91
+26	87	151	− 56
+25	127	155	− 28

The discordance thus increases in the middle. Now there is undoubtedly a local obscuration of special intensity in the Oxford zones, as we have known from the very earliest plates taken near 4^h, and as is plainly shown in all our results. [It attracted the attention of Melotte and Dyson, who gave a map of it from the Franklin Adams charts in *M.N.*, **80**, 3.] This feature might either be a part of the more general spiral or in addition to it; and the above figures favour the latter interpretation. The maximum of the local obscuration is at about R.A. 4^h or 60°, and it seems to drag the maximum away from the 110° or 127° shown by the outer zones.

7. This differential effect (as between bright and faint stars) does not, however, do full justice to the facts, which are better shown in Tables VII. to XIII. We see how persistently negative are the coefficients of sin θ, for instance: and the same is true of other coefficients, even of the fourth harmonic, on which a few remarks may be made before returning to the first. It will be seen in Table XIV. that its rates of change are curiously small: the separate zones differ sensibly, but their mean values show no change as we pass from bright stars to faint. It has been also noticed that there is a striking absence of change as we pass from one declination to another even to the southern hemisphere. The fourth harmonic, in fact, seems to represent some real feature of stellar distribution. As regards the present material alone, accepting the fact of no change from bright stars to faint, we may take the mean result for each zone so as to reduce accidental errors; and to give a standard of comparison we will do the same for the third harmonic.

TABLE XV.

Mean Third and Fourth Harmonics.

Zone.	sin 3θ.	cos 3θ.	Max.	O–C.	sin 4θ.	cos 4θ.	Max.	O–C.
+31°	−·011	+·025	336°	−105°	+·056	−·009	99°	+19°
+30	+·014	−·034	158	+ 77	+·066	−·006	95	+15
+29	+·095	+·004	67	− 14	+·054	−·027	116	+36
+28	+·056	−·023	113	+ 32	+·050	+·046	47	−33
+27	+·038	+·031	50	− 31	+·135	+·051	69	−11
+26	−·010	−·025	202	+121	+·058	+·040	56	−24
+25	−·041	+·040	314	−127	+·065	−·010	99	−19
	+·020	+·003	81	..	+·069	+·012	83	

The third harmonic shows no constancy, but the phase of the fourth clings pretty closely to its mean value.

8. Returning now to the first and second harmonics, the rates of change are larger, and we shall not gain much by taking means for the whole zone (see especially the first harmonic for zone +25°, where the bright stars differ notably from faint). But we may take means of the last three groups in each zone as in Table XVI.

TABLE XVI.

Means of Last Three Rows for First and Second Harmonics.

	sin θ.	cos θ.	Min.	Diff.	sin 2θ.	cos 2θ.	Max.	Diff.
+31°	−150	−079	62°	+ 8°	−·088	+·023	285	− 7°
+30	−252	−128	63	+ 9	−·096	−·061	238	−54
+29	−060	−065	43	−11	−·069	+·072	316	+24
+28	−132	−174	37	−17	−·105	+·074	305	+13
+27	−077	−069	48	− 6	−·098	+·071	306	+14
+26	−144	−075	63	+ 9	−·081	+·012	278	−14
+25	−065	−054	50	− 4	−·071	+·030	293	+ 1
Mean	−·126	−·092	52		−·087	+·034	289	

For the first harmonic the minimum is tabulated (as equivalent to a maximum *obscuration*): it will be seen that it lies pretty steadily at R.A. 54° for all the seven zones. For the second harmonic the maximum 292° represents all the zones but +30° pretty closely. Further comment is reserved until zones in all parts of the sky are presented together.

Summary.

Counts are given for all the Oxford zones +31° to +25° and analysed by the standard process of *M.N.*, **78**, 55. The rate of change (from bright to faint stars) of the first harmonic does not now accord with

the spiral formerly suggested ($\alpha+3{\cdot}66=247°$) : δ but the first, second, and fourth harmonics for all stars are large and consistent for the different zones, indicating definite features of stellar distribution distinct from the galactic condensation in latitude (which was removed before the formation of these figures). Further discussion is reserved until various regions of the sky are presented together.

University Observatory, Oxford :
1925 *March* 9.

Orbit of the Double Star Σ1037=*B.G.C.* 3876.

By Geo. Van Biesbroeck.

This double star, which is B.D.+27° 1337 at $7^h\ 9^m\ 43^s$, +27° 19′ (1950), was first measured by W. Struve in 1827. It consists of two equal components of magnitude 7·2 and spectral type F5. After slowly separating until 1860 the components gradually closed in and recently periastron has been passed, so that a fairly sharp determination of the elements is now possible.

In 1917 Russell made a first investigation of the orbit (*Publ. Amer. Astr. Soc.*, **3**, 328). He concluded that the observations could be represented by orbits ranging from 150 to 510 years. The *Greenwich Catalogue of Double Stars*, published in 1921, contains (p. 169) the following remark about this pair: "A minimum separation of about 0″·12 should have been reached at position angle 200° about 1919·5, and the companion should pass into the first quadrant about 1925. The period will be about 200 years, the eccentricity about 0·8, and the inclination large."

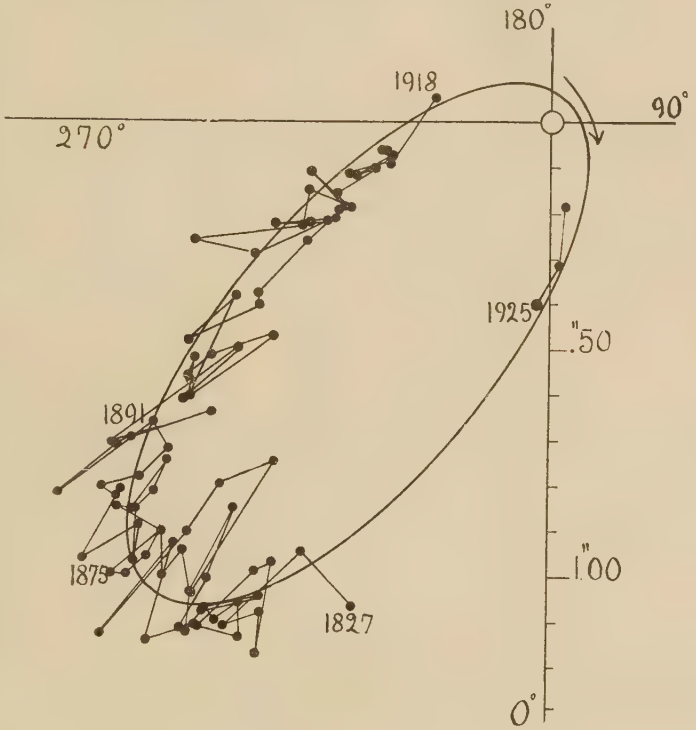
The following observations, both positive and negative, obtained here with the 40-inch refractor, show a motion unexpectedly faster, with a correspondingly shorter period and higher eccentricity :

1916·01	280°·0	0″·36	2*n*
18·37	259·2	0·25	4
20·18	not separated <0″·15		
20·97	doubtful elongation in 90° ? ±0″·1		
21·14	not separated <0″·15		
23·08	9·3	0·19	1*n*
24·37	3·3	0·33	3
25·08	357·0	0·40	5

The quadrant is, of course, indeterminate, but the one adopted is the only way of satisfying the law of areas. Thus the companion is one quadrant ahead of the expected position. It is now in the fourth quadrant, so that the total change is only some 20 degrees short of a revolution.

In order to determine the orbit, all published measures, together with the above ones (Table I.), were first plotted against the time. The graphically interpolated curves of the angles (reduced to 1900) and the

distances were mutually checked so that the areal velocity would remain constant. This check was especially valuable for the earlier measures, which show a large range. The apparent curve for the period 1827–1918 could now be drawn. The missing part of the orbit was completed graphically, and the constancy of the areal velocity controlled also in that part by the planimeter. This indicated that at the periastron 1920–21 the minimum distance became as small as 0″·05. The figure shows the curve obtained. All the measures from Table I. have been plotted in this figure in order to illustrate their scattering.



Apparent orbit of Σ1037.

The elements were deduced by Zwiers' method. Without further changing the dynamical elements, those defining the position of the orbit were slightly adjusted so as to remove some minor outstanding deviations between the run of the observed and the computed values. The material is of such a nature that a least-square adjustment would hardly justify itself. The measures by some of the observers are evidently affected by large systematic errors which cannot easily be eliminated. The corrections finally adopted were found by empirically changing ω and i until the values of $\theta_0 - \theta_c$ became constant. The residuals in angle and distance are somewhat contradictory in the recent observations. If the first are removed the distances are syste-

matically too large, while if the distances are represented closely there are one-sided deviations in the angles. The differences are small, however, and are probably to be attributed to the preponderance of the same observer at the end of the list. The whole determination is, of course, of a preliminary character, and it will probably have to be left so until the remaining part of the orbit is covered. The elements finally adopted are :

$T = 1920{\cdot}57$
$e = 0{\cdot}932$
$\Omega = 31^\circ{\cdot}4$
$i = 39^\circ{\cdot}0$ } 1900
$\omega = 254^\circ{\cdot}1$

$a = 0''{\cdot}870$
$n = 2^\circ{\cdot}991$ (P = 120·4 years)
motion retrograde

The eccentricity is one of the largest known. The only decidedly higher value is 0·96, deduced for Σ1865 by Hertzsprung. Of the same order are the eccentricities of Σ2525 (0·933) and of Σ1639 (0·926) computed by Jackson (*Greenwich Cat.*, pp. 224 and 212 resp.).

It is of interest to verify if these elements account for my failure in trying to see the star double in 1920 and 1921.

For the three nights considered the computed positions are :

	°	″
1920·18	215·7	0·08
20·97	60·7	0·08
21·14	47·3	0·10

It is therefore not surprising that the stars could not be separated. Only on the last of the three dates a slight elongation might have been visible if the atmospheric conditions had been perfect, which was not the case. The elongation suspected on the second date seems real, but this estimate is too uncertain to be introduced into the computation of the elements. The negative results obtained here in 1920–21, therefore, independently confirm the adopted elements.

TABLE I.

Date.	Angle Obs.	Dist. Obs.	O–C.		Observer.	No. of Nights.
	°	″	°	″		
1827·28	337·8	1·15	+ 3·4	+0·05	W. Struve	2
32·00	330·2	1·10	− 1·8	−0·07	,,	4
36·28	327·4	1·32	− 2·3	+0·10	,,	3
40·28	329·7	1·25	+ 1·2	−0·01	O. Struve	3
41·80	331·1	1·33	+ 3·0	+0·05	Maed.	3
42·16	328·0	1·14	+ 0·1	−0·14	Glaisher	2
43·17	326·8	1·18	− 0·7	−0·11	Dawes	4
43·20	325·2	1·35	− 2·3	+0·06	Kaiser	8–3
43·23	329·2	1·32	+ 1·8	+0·02	Maed.	2
45·29	327·0	1·26	+ 0·3	−0·04	,,	6
46·30	326·2	1·32	− 0·1	+0·02	,,	18
47·60	324·6	1·31	− 1·2	0·00	,,	3
47·74	328·7	1·22	+ 2·9	−0·09	O. Struve	2
48·17	324·7	1·32	− 0·5	0·00	Dawes	1

TABLE I.—*continued.*

Date.	Angle Obs.	Dist. Obs.	O–C.		Observer.	No. of Nights.
	°	″	°	″		
1851·65	324·8	1·37	+ 0·4	+0·04	Maed.	18
55·39	323·1	1·30	+ 0·1	−0·04	,,	18
55·40	320·7	1·10	− 2·3	−0·24	De.	8
56·67	323·1	1·25	+ 1·4	−0·09	Secchi	2
59·24	324·0	1·37	+ 2·3	+0·03	Maed.	6
60·11	325·0	1·35	+ 3·6	+0·01	Morton	3
60·55	322·4	1·44	+ 1·1	+0·10	Maed.	4
63·20	318·2	1·24	− 2·1	−0·10	De.	4
65·22	318·8	1·49	− 0·8	+0·16	Engel.	4
65·62	318·5	1·20	− 0·9	−0·13	De.	2
67·21	317·7	1·07	− 1·2	−0·25	Kaiser	6
67·66	320·8	0·96	+ 2·1	−0·36	Searle	2
68·23	322·8	1·30	+ 4·3	−0·01	O. Struve	3
71·24	319·4	1·24	+ 2·0	−0·06	,,	3
71·92	319·5	1·31	+ 2·4	+0·01	Duner	3
72·16	316·9	1·40	− 0·1	−0·10	Wilson and Seabr.	1
72·25	316·4	1·24	− 0·6	−0·05	De.	1
72·26	315·9	1·38	− 1·1	+0·09	Tidblom	1
73·07	317·0	1·30	+ 0·3	+0·01	Brunnow	1
73·87	316·9	1·36	+ 0·5	+0·08	Wilson	2
74·13	317·1	1·30	+ 0·8	+0·02	Gled.	1
75·11	316·6	1·24	+ 0·7	−0·03	Duner	3
75·19	317·6	1·31	+ 1·8	+0·04	Seabr.	1
76·15	311·6	1·27	− 3·9	+0·01	Hall	3
77·73	312·8	1·25	− 2·1	0·00	Schiap.	4
79·18	311·5	1·12	− 2·8	−0·12	P. Smith	1
79·25	312·9	1·19	− 1·4	−0·05	O. Struve	2
80·22	313·0	1·24	− 0·9	+0·01	Seabr.	2
80·23	316·5	1·33	+ 2·6	+0·10	Dob.	2
81·23	314·4	1·27	+ 1·0	+0·05	Jed.	6
82·82	313·2	1·40	+ 0·5	+0·20	Engel.	7
84·18	310·6	1·24	− 1·5	+0·06	Perr.	2
87·18	310·9	1·26	+ 0·1	+0·12	Tarrant	2
87·27	309·1	1·27	− 1·6	+0·13	H. Struve	2
87·27	311·3	1·19	+ 0·6	+0·05	Jed.	4
89·20	310·6	1·10	+ 0·9	−0·01	Hall	3
91·34	305·3	1·18	− 2·8	+0·10	Robinson	2
91·45	307·1	1·09	− 1·4	+0·02	Big.	3
91·52	307·1	1·35	− 1·3	+0·28	Collins	4
92·76	306·6	1·18	− 1·3	+0·13	Knorre	2
93·05	310·6	0·98	+ 2·9	−0·07	Glase.	2
93·08	307·5	1·19	− 0·2	+0·15	Jones	2

TABLE I.—*continued*

Date.	Angle Obs.	Dist. Obs.	O–C.		Observer.	No. of Nights.
	°	″	°	″		
1893·24	306·9	1·07	− 0·7	+0·03	Maw	3
94·12	306·2	1·19	− 0·8	+0·16	Big.	1
94·13	306·0	0·85	− 1·0	−0·18	Holmes	2
96·08	307·0	1·01	+ 1·2	+0·02	Collins	2
96·19	307·8	0·77	+ 2·1	−0·21	Coleman	4
97·00	304·5	0·91	− 0·7	−0·06	Greenw.	13
97·25	304·8	0·98	− 0·2	+0·02	Morgan	2
98·08	303·3	0·94	− 1·1	−0·01	Hussey	4
98·94	305·8	0·98	+ 2·0	+0·05	Seabr.	3
99·25	307·1	1·00	+ 3·5	+0·08	Maw	1
1901·94	299·0	0·79	− 2·4	−0·07	Greenw.	14
02·51	301·5	..	+ 0·6	..	Postelm.	3
03·02	301·1	0·93	+ 0·6	+0·10	Dob.	3
03·24	302·1	0·76	+ 1·8	−0·07	V. B.	3
04·15	300·4	0·75	+ 1·0	−0·05	,,	3
06·30	296·4	(1·31)	− 0·7	..	Ichinoe	3
06·79	295·8	0·60	− 0·7	−0·12	Greenw.	12
07·99	293·9	0·54	− 1·1	−0·15	V. B.	5
08·22	292·4	0·69	− 2·5	+0·01	Przyb.	4
09·20	289·8	0·65	− 3·5	0·00	Dob.	3
09·20	293·9	0·52	+ 0·6	−0·13	V. B.	3
10·08	294·1	0·72	+ 2·4	+0·11	Dool.	3
10·26	288·2	0·83	− 3·4	+0·22	Dob.	2
11·11	292·2	0·59	+ 2·1	+0·01	V. B.	2
11·12	285·9	0·56	− 4·2	−0·02	Dob.	1
11·18	293·0	0·48	+ 3·0	−0·10	Greenw.	11
12·16	281·9	0·54	− 6·1	0·00	Wirtz	1
12·64	292·3	0·49	+ 5·8	−0·03	Dool.	3
13·08	288·4	0·50	+ 2·6	−0·01	Voute	4
13·12	284·5	0·40	− 1·2	−0·11	V. B.	3
13·96	285·5	0·45	+ 2·1	−0·02	Greenw.	10
14·11	281·6	0·36	− 1·4	−0·10	V. B.	2
14·12	291·9	0·51	+ 8·9	+0·05	Rabe	4
14·25	284·1	0·46	+ 1·5	+0·01	Dob.	2
15·24	283·3	0·37	+ 4·1	−0·04	V. B.	3
15·24	279·7	0·38	+ 0·5	−0·03	Rabe	4
16·01	280·0	0·36	+ 4·0	−0·01	V. B.	2
17·70	283·6	0·37	(+17·8)	+0·09	Dob.	2
18·37	259·2	0·25	− 0·2	0·00	V. B.	4
23·08	9·3	0·19	− 1·4	−0·08	,,	1
24·37	3·3	0·33	0·0	−0·05	,,	3
1925·08	357·0	0·40	− 3·6	−0·02	,,	5

In Table I. the individual yearly means have been compared with the computed values. The residuals in angle show that the whole series of measures is represented as well as can be expected. This comparison was made especially to see if there is any foundation for J. Maedler's hypothesis of an invisible third component, by which he tried to account for the large deviations in the measures by W. Struve and himself up to 1846 (*Untersuchungen über Fixstern-Systeme*, Erster Theil, 1847, p. 258). Maedler observed the star very frequently, making, for instance, measures on eighteen nights in 1846, 1851, and 1855. He convinced himself that there were oscillations up to 5° in the angles and 0″·25 in the distances. By elaborate computations he represented these by the following elements :

$T = 1827{\cdot}72$
$e = 0{\cdot}6316$
$\Omega = 156°\ 58'$
$\omega = 273°\ 27'$ } 1845
$i = 68°\ 17'$
$a = 0''{\cdot}1817$
P = 15 years'
motion direct

How illusory these refined computations are can be seen by a glance at the residuals in Table I., in which no 15-year oscillation running up to 5 degrees is noticeable. There are large residuals, to be sure, but not of a periodic character. If Maedler had had at his disposal the early measures by Dawes, Kaiser, and Glaisher, he would have noticed that the regular run indicated by his observations is not supported by the measures of other observers.

In Table II. is given for future comparison an ephemeris computed from my elements (apparent angles) :

Table II.

	°	″	
1924·0	6·0	0·36	5·7 km/sec.
26·0	357·6	0·47	4·8 ,,
28·0	352·9	0·57	4·2 ,,
30·0	349·4	0·65	3·8 ,,
32·0	346·7	0·72	3·4 ,,
34·0	344·5	0·79	3·2 ,,
36·0	342·6	0·85	2·9 ,,
38·0	341·0	0·90	2·7 ,,
1940·0	339·4	0·95	2·5 ,,

The star is now coming again within the reach of telescopes of moderate size.

The only direct determination of the parallax of this star was obtained at the Sproul Observatory : +0″·014, ±0″·011. The hypothetical parallaxes, computed in different ways from the curvature of the orbit and under the assumption that the sum of the masses of the two components is equal to twice that of the sun, vary from 0″·018 to 0″·043 (*Greenwich Cat.* of D. St., p. 169). Under the assumption of a mass of

1·5 for each component, which is the value to expect for a dwarf F5 star, our elements lead to a parallax of 0″·039, or 83 light years.

This would make the semi-major axis of the orbit 30 astronomical units, and the minimum separation at periastron only 2·0 units. The orbital velocity at that time was such that the Doppler shift in the spectrum must have produced a measurable duplicity of the lines. The relative velocity resulting from the above elements is :

$$\pm V = -2{\cdot}4 + 9{\cdot}5 \cos (v+\omega) \text{ km/sec.}$$

It passed from the extreme value −11·9 km/sec. in 1919·62 to the other extreme of +7·1 km/sec. in 1921·00. It is now no longer measurable, as can be seen from the values given in Table II.

This pair has a distant companion of mag. 13, which has been measured as follows relatively to the mean of A and B :

1899·95	110°·6	16″·32	Hussey	3*n*
1925·07	103°·9	15″·75	V. B.	2*n*

Assuming C to be fixed, the change in 25 years would indicate for AB a p.m. of 0ˢ·0000 and −0″·076. The *Greenwich Catalogue* for 1910 gives +0ˢ·0004 and −0″·090 from meridian observations. From this it appears that C does not belong to the system.

Miss M. Jorgensen helped efficiently in the computations required by this investigation.

Yerkes Observatory, Williams Bay, Wis. :
1925 *February* 17.

Observations of β Lyræ by Members of the B.A.A., 1906–1920.

Edited by Mary A. Blagg.

[*Note.*—The unit employed throughout this paper is 100th of a magnitude, except when otherwise indicated, the object being to dispense with the continual use of the decimal point.]

The observations of β Lyræ made by members of the B.A.A. during the years 1906–20 have been most kindly entrusted to me by Colonel Markwick, that they might be investigated with a view to testing the permanent existence of the secondary variation discovered through Baxendell's observations of 1840-77 (*M.N.*, **84**, 8). The B.A.A. observations were taken by nine different observers, but in the case of five of these their observations were too few or too much scattered to allow of the possibility of ascertaining their personal peculiarities. In such a close investigation as this it is of the first importance to co-ordinate the observations as far as possible, unless those of each observer are dealt with independently and a separate mean curve

formed for each. In this case the latter plan is not feasible, the observations of each individual observer being insufficient alone for a satisfactory mean curve.

The four observers whose work was finally used are Colonel Markwick, Professors Mitchell and Moye, and Monsieur F. de Roy, henceforth deignated Ma, Mi, Mo, and Ry ; and the number of observations used is 1500. Ma's observations extend from 1906 to 1920, omitting 1915 ; Mi's from 1906 May to 1907 Jan. 3 only, but are fairly regular up to October ; Mo's from 1906 Oct. to 1919 Nov. ; Ry's from 1906 June to 1909 Oct. The same comparison stars are used by all, but the times of observation and methods of comparison vary a good deal. Almost all were naked-eye observations.

Such of the deduced magnitudes as had been taken to the nearest 10th magnitude only were reduced to the nearest 100th, and all the observations of the nine observers were tabulated according to their position in the cycle, using the formula given in *B.A.A.J.* (**33**, No. 7, 281–4) for position of principal minimum, and dividing the cycle into 12 equal portions. This formula places the epoch later than does the one which I deduced from Baxendell's work, but up to the date at which his observations ended the difference would be insignificant. Using the Baxendell mean curve as the standard for comparison, the residual for each individual observation of all observers was obtained (subtracting 10 from Ma's and adding 11 to Ry's), up to the end of 1909, with a view to finding (*a*) any necessary corrections to the mean curve, and (*b*) evidence as to the accuracy of the assumed position of the minimum ; both highly important points.

The result showed little difference in the form of the curve, the chief alteration being a greater general brightness throughout (except at the principal minimum), greatest near the secondary minimum, and small at maximum.

Calculations for (*b*) were made both with and without the application of these corrections to the curve and residuals, the results being practically identical. There was an obvious marked difference between the residuals from the rising slopes of the curve and those from the descending slopes. Taking each side of the principal minimum, the general difference was : (rise−fall)=−3, and each side of the secondary one : (rise−fall)=$-3\frac{1}{2}$; a clear indication that the minimum was placed too far on. A calculation, using all the 12 portions, weighted by $(p-f)$, gave a correction to epoch of −0·07 days. As a further test the observations of the three chief observers (Ma, Mo, and Ry) up to the end of 1911 were *separately* tabulated, without applying any personal corrections ; the means of each set were calculated for each portion of the curve ; and then the mean of these three means was taken. (The number of observations from the observers did not greatly differ, those of Mo being rather the most numerous and those of Ma the fewest.) These mean figures were dealt with as the others had been, and the resulting correction to epoch was −0·094 days. Treated in a different manner, using fractions of a day directly, instead of residuals of magnitude, the first set of figures gave the correction −0·10 days ; and the second set, dealt with by yet another (analytical) method,

gave −0·099 days. From these results it seemed fairly safe to conclude that the B.A.A. formula requires a correction (for 1908 or 1909) of about −0·1 days. On looking at the curve given in *B.A.A.J.* (see above), it was seen that the lowest point in the black-dot variation would actually occur about halfway from the end of the last division, which division represents 0·42 days, thus suggesting a correction of −0·2 days about 1913. The open-dot curve does not show this feature, but that is formed from considerably earlier observations, at the date of which this correction would not be noticeable. As the formula deduced from Baxendell's observations would give far too large a correction, a new formula was evolved. One which was first tried did not go far enough, and a second one went slightly too far. The one finally adopted is given below :

1. Baxendell : $=2398590{\cdot}57 + 12{\cdot}908E + 0{\cdot}0000037E^2$.
2. *B.A.A.J.* : $=2398590{\cdot}604 + 12{\cdot}908006E + 0{\cdot}000003946E^2$.
3. Now adopted : $=2398590{\cdot}57 + 12{\cdot}908006E + 0{\cdot}000003914E^2$.

The correction to the B.A.A. Epoch near the end of 1913 works out to −0·13 days, and in 1906–8 to −0·10 days.

E=1442 to 1864 in the course of these observations.

[After the initial dates of all the cycles had been corrected in accordance with this 3rd formula and the residuals all retaken, the final figures were dealt with as before. The correction to epoch then obtained from all the observations (corrected for personality) of the four observers, 1906–20, with this new formula, is less than −0·001 days, which is too small to have any significance.]

The mean curve found as stated above was, of course, drawn smoothly and symmetrically, while adopting as nearly as possible the actual mean values obtained from the observations. Table I. gives the 12 points taken, and also the figures for the 4 epochs. Some intermediate points were also taken.

TABLE I.

Mean Curve.

Points Mag. (×100)	1 & 12	2 & 11	3 & 10	4 & 9	5 & 8	6 & 7	Min. 1	Min. 2	Max. 1 & 2
	395	362	352	352	360	373	410	378	351

The next question in importance is that of personal differences between the four observers. The most obvious of these was the mean difference of about 21 (0·21 m.) between Ma and Ry. But the difference was not constant throughout the whole range, being very much greater in the fainter than in the brighter part. Ma's total range in deduced magnitudes was by far the widest of the four (and also far wider than Baxendell's), especially in the later years, extending through 1·7 magnitude. That of the other three extended through about 0·95 magnitude, in fairly close agreement, and Baxendell's was about the same as theirs. During the first few years Ma frequently used several comparison stars, and his range did not then greatly differ from the

others. But after 1910 he confined himself mainly to comparisons with γ Lyræ, so that his intervals were often very wide, and it seems certain that his " steps " with the naked eye were smaller than tenths of a magnitude. (This is borne out by his remarking on several occasions that, seen through binoculars, the interval between β and γ appeared less.) Four " steps " seem to be roughly equal to 0·3 magnitude. After a great deal of calculation, both individually and in combination, the following table was evolved :

TABLE II.

Personal Corrections.

Observer.	Limits.	Correction.
Ma	Throughout	$-0{\cdot}25\,(m-340)$
Mi	,,	None
Mo	$m \geqslant 420$	$+0{\cdot}29\,(372-m)$
,,	$m > 420$	$-10-0{\cdot}4\,(430-m)$
Ry	$m \geqslant 390$	$+15-0{\cdot}1\,(390-m)$
,,	$m > 390$	$+12-0{\cdot}4\,(m-400)$

m=the unaltered deduced magnitude of the observation.

It is not claimed that these corrections bring the four observers into actual agreement in detail—an impossible task. But they bring about as near an approach to that result as I believe to be feasible. It must be admitted that they slightly reduce the total range of variation, but this does not matter for my purpose, so long as the mean of the observations in each portion is in agreement with the mean curve as drawn. My concern is with the residuals, and the most likely effect it could have upon these would be to reduce them slightly in amplitude—an error on the safe side. Ma's general mean of (nominally) +1 (see below) suggests that his correction may not go quite far enough, but the mean is actually much less than 1. Without a knowledge of the details, Mo's correction might be suspected of unduly reducing his deduced magnitudes near minimum, but this is not really the case. He *very* rarely observed the star close to its minimum, and on each of such occasions his corrected observations fit in well with the others.

These preliminaries accomplished, the residuals, corrected by Table II., were examined with a view to discovering whether the two variations (A) and (B) found in Baxendell's work would appear here also. As it was not intended to *apply* the " Seasonal " corrections (A) (if any), the residuals were first merely tabulated in two divisions, " S " from April to August 15, and " W " from August 16 to January ; inspection having shown an apparent change in August or thereabouts. (The star souths near the usual observation hours in August.) February and March were omitted as having few observations and being perhaps transitional. January was added to the preceding year.

TABLE III.

Seasonal Variation (A).

Observer.	Date.	Mean Residuals. S.	W.	S–W.	No. of Observations.
Ma	1906–08	+ 3	− 3	+ 6	131
,,	1909–12	+ 5	− 2	+ 7	119
,,	1913	− 4	− 3	− 1	72
,,	1914–18	+12	+ 1	+11	117 (S only 34)
,,	1919–20	+ 4	− 5	+ 9	111
,,	All Ma	+ 5	− 2	+ 7	550
Mi	1906 (all)	+ 1	− 1	+ 2	60
Mo	1906–08	+ 9	− 2	+11	149
,,	1909–11	+ 7	− 1	+ 8	125
,,	1912–14	− 1	+ 5	− 6	127
,,	1915–17	+ 1	− 3	+ 4	157
,,	1918–19	− 1	−10	+ 9	90
,,	All Mo	+ 3	− 2	+ 5	651
Ry	1906–07	+ 2	− 3	+ 5	93
,,	1908–09	+ 2	− 6	+ 8	130
,,	All Ry	+ 2	− 4	+ 6	223
All	Total	+ 3	− 2	+ 5	1484

General mean : Ma, +1 ; Mi, 0 ; Mo, 0 ; Ry, 0.

The year 1913 is given separately for Ma, as it forms the only exception to the general rule in his case. In Mo's case, 1912 and 1914 share in this exception.

For the rest, the seasonal effect is clearly marked and fairly consistent, considering that it is of course not a *direct* effect of the seasons, and also that some of the observations were taken in England and some in France, and that the hours of observation often differed greatly.

The general result here does not accord with that of Baxendell. His residuals, when arranged according to months, gave positive means from August to November and negative from December to July, and it was found that they depended systematically on orientation and altitude. It looks as though the orientation effect were here reversed, and there are some tokens that the lightness or darkness of the sky was an important factor, *e.g.* observations in strong moonlight or twilight are wont to give abnormally large positive residuals, in agreement with the effects of S and W. But, marked as is this seasonal variation in Table III., it is not sufficiently consistent, nor are the causes sufficiently certain, to admit of its being formulated and applied, without more expenditure of time and labour than appeared to be desirable.

We come now to the secondary variation (B) which was found in Baxendell's work, and the testing of which was the main object of the present investigation. The mean period of this wave in the Baxendell observations was found to be rather over 6·576 days, with an apparent fluctuation of the epoch of minimum, backwards and forwards, amounting to 1·22 days each way, in a period of 3562·608 days (between 9 and 10 years).

An exactly similar plan of procedure has been followed in this case, but the period assumed for the purpose of investigation was 6·57 days; a little less than the Baxendell mean. A series of short groups of cycles of this length was taken, on the principle of obtaining an example wherever there was a sufficient number of observations, not too much scattered, and without much variation in the effect of (A). Regard was also had to getting as great variety as possible in respect of the observers. No observations within the limits of a group were rejected except those indicated as doubtful by the observer, those (very few) which gave a corrected residual of half a magnitude or more (almost certainly mistakes), and three very isolated observations, one (by Mi) in No. 3, and the others in No. 4. But in some of the examples from one observer only, a few observations by some other observer were available which were not included in order to keep the example entirely to the one.

The interval between each group and the next is always an exact multiple of 6·57 days. Table IV. shows the results obtained, and is arranged on precisely the same lines as Table VII. in my Baxendell paper, except that here two additional columns give the equivalent of O − CF in days, and details as to the observers. The number of cycles in a group varies from 4 in No. 25 to 14 in No. 22.

The column N gives the number of cycles of 6·57 days elapsed from the mean of group No. 7 to the mean of each group in succession. The next 6 give the mean residual in each of the 6 equal subdivisions of the cycle; and it will be seen that they support in every case, without exception, the genuineness of this variation. The next column gives the number of observations in each group, and the next the position of the minimum, harmonically analysed in the usual way. Where the observations are scanty or irregular there is considerable room for error in the analysis, since each division contains 60°, but the general trend of the figures is unmistakable.

If the mean period were the same as in the Baxendell observations there should be an average advance in phase of a little over 0·006 days (=0°·33) per cycle. But this column shows clearly that the advance is greater than that. By calculation, by graphical methods, and by trial as to least squares, it was found to be +1°·38 per cycle, equal to just over +0·025 days, thus making the mean period of this variation about 6·595 days, or 0·019 days longer than before. It was suggested in the Baxendell paper that there was a possibility that the period was slowly increasing, and now we find that during the interval between the two series it apparently *had* increased by the above amount.

The column C_1 gives the phase obtained by adding 1°·38N (less any complete multiples of 360°) to 208°, which is nearly the observed phase when N=0. The following column gives $O-C_1$. The average

TABLE IV.

	N.	(1).	(2).	(3).	(4).	(5).	(6).	No. of Obsns.	Position of Min.	C_1.	$O-C_1$.	CF.	O-CF.	Equivalent in Days.	Observers.
									°	°	°	°	°	d	
1	−109	+ 6	− 4	− 4	− 3	− 1	+ 9	21	315	417	−102	−81	−21	−0·383	Ma, Mi, and Ry.
2	− 98	+ 3	− 1	− 7	− 5	+ 3	+ 7	41	310	433	−123	−79	−44	−0·803	,,
3	− 86	+ 5	− 3	− 4	− 6	− 6	+ 3	49	348	449	−101	−76	−25	−0·456	All four.
4	− 62	+14	+ 5	− 3	− 8	0	+ 9	17	354	482	−128	−64	−64	−1·168	Ry.
5	− 50	+ 1	+ 8	+ 8	+ 9	+ 4	+ 2	46	136	139	− 3	−56	+53	+0·967	Ma, Mo, and Ry.
6	− 35	− 7	− 1	+ 6	+ 7	+ 1	− 4	22	161	160	+ 1	−44	+45	+0·821	Ma and Ry.
7	0	− 5	− 5	+ 2	+ 9	+ 9	− 1	21	203	208	− 5	−11	+ 6	+0·110	,,
8	+ 23	− 3	− 5	− 9	− 3	− 2	− 1	54	285	240	+ 45	+12	+33	+0·602	Ma, Mo, and Ry.
9	+ 54	+ 7	0	− 3	0	+ 3	+10	27	311	283	+ 28	+41	−13	−0·237	Ry.
10	+ 65	+ 9	+ 3	− 4	− 5	− 3	+ 8	9	347	297	+ 50	+50	+ 0	0·000	Ma.
11	+ 82	+11	+ 7	− 3	−15	−10	+ 6	32	370	321	+ 49	+62	−13	−0·237	Ma, Mo, and Ry.
12	+120	+15	+22	+16	+15	+12	+ 7	35	88	14	+ 74	+79	− 5	+0·091	Ma and Mo.
13	+135	− 6	− 3	0	− 6	− 9	− 6	17	90	34	+ 56	+81	−25	−0·456	Mo.
14	+172	−12	− 6	+ 2	+ 7	+ 6	+ 3	21	205	85	+120	+73	+47	+0·858	,,
15	+228	−16	−18	+ 4	+14	+13	+ 3	19	209	163	+ 46	+34	+12	+0·219	,,
16	+246	− 2	− 7	+ 6	+ 6	+ 7	+ 2	24	207	188	+ 19	+17	+ 2	+0·036	Ma and Mo.
17	+283	− 6	−10	− 7	− 1	+10	− 5	39	234	239	− 5	−20	+15	+0·274	,,
18	+298	− 2	− 8	− 7	− 6	− 1	+ 3	25	290	259	+ 31	−34	+65	+1·186	Ma.
19	+344	+ 1	−17	−20	+ 3	+16	+14	23	269	323	− 54	−69	+15	+0·274	Mo.
20	+397	+ 3	−13	−12	+ 3	+ 6	+10	25	272	396	−124	−81	−43	−0·785	,,
21	+469	−14	− 7	+ 9	+ 2	−10	−17	20	138	135	+ 3	−44	+47	+0·858	,,
22	+518	− 5	− 9	− 5	+ 5	+ 7	+ 1	41	232	203	+ 29	+ 3	+26	+0·474	,,
23	+567	+ 6	− 2	− 6	− 3	+ 5	+ 6	48	305	271	+ 34	+49	−15	−0·274	Ma and Mo.
24	+630	+ 4	+ 2	− 2	− 6	−23	−12	21	60	−2	+ 62	+80	−18	−0·328	Ma.
25	+679	− 5	+17	− 4	− 8	−21	−10	10	64	65	− 1	+72	−73	−1·332	,,

residual is numerically smaller than in the corresponding column for Baxendell, but there are a greater number of 3-figure, as well as more single-figure, residuals. There is also a quite equally strong suggestion of a fluctuation, which, however, appears to be shorter than the one assumed in the other series. Its probable length was found to be $504\times6\cdot57$ days$=3311\cdot28$ days, just over 9 years. (That in the previous series had been taken as $552\times6\cdot454$ days$=3562\cdot608$ days, but it was mentioned that rather shorter periods gave almost equally good results.) Harmonic analysis and the least-square test gave the formula $81^\circ\cos(\theta-98^\circ)$, and the column CF gives the resulting correction for the position of the minimum phase for each group. The complete formula for correction would thus be

$$+1^\circ\cdot38\mathrm{N}+81^\circ\cos(\theta-98^\circ) \text{ where } \theta=360^\circ\mathrm{N}/504 \text{ or } \tfrac{5}{7}\,\mathrm{N}^\circ.$$

The fluctuation formula in the Baxendell series was $68^\circ\cos(\theta+35^\circ)$, where θ equalled about $0^\circ\cdot652\mathrm{N}$. But the degree has a different value in days in the two series, so in order to compare them the equivalent in days must be given. The Baxendell mean period was nearly 6·5764 days, and the fluctuation in the phase amounted to 1·22 days each way in an assumed period of 3562·608 days. The B.A.A. period is nearly 6·5952 days, and the fluctuation in the phase amounts to nearly 1·48 days each way in a period of 3311·28 days. The correspondence between these figures is near enough (considering the circumstances) to form a really strong corroboration of the main facts, while it is not close enough to enable us to fix precise values.

The next two columns in Table IV. give O−CF in degrees and also in days. The mean agreement is distinctly closer than in the Baxendell series, three-fifths of the residuals being under $\pm30^\circ$ (0·55 days) and all except three under $\pm60^\circ$ (1·1 days). The average residual is $\pm29^\circ$. There is, however, still an appearance of system about the arrangement of these residuals, which might indicate a second, shorter fluctuation, or possibly that the mean period is now growing slowly *shorter*. The increased mean length of the period since Baxendell's time is probably a reality, but the fluctuation formulæ are both uncertain, and it is doubtful whether there is any real change in that respect, though the evidence for a fluctuation in something over nine years is greatly strengthened.

The final column gives the observers responsible in each group. One is by all 4, 5 by 3, 6 by 2, and 13 by a single observer, 3 different ones being represented in these.

An attempt was now made (as was done in the Baxendell paper) to obtain the mean form and amplitude of this variation. Each example was plotted on squared paper, curves were drawn smoothly through the dots, and intermediate points read off between each of those already given. In Table V. these are arranged one below another, shifted from their positions in Table IV. in such a manner as to make their phases coincide as nearly as possible. It was again necessary to have two divisions, but a few of the more irregular examples are so ambiguous as to phase that it was difficult to know how to place them. The question was decided in these cases by calculating the balance of the figures.

TABLE V.

The Examples in Table IV. collated.

No.	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).	(10).	(11).	(12).	Date.	S or W.	Mean Resid.
							(a) Examples with Max. and Min. nearly at a point.								
7	− 1	− 2	− 5	− 6	− 5	− 2	+ 2	+ 7	+ 9	+10	+ 9	+ 6	1908 May and June	S	+ 2
8	− 4	− 5	− 8	− 9	− 7	− 5	− 3	− 2	− 1	− 1	− 2	− 3	,, Sept. to Nov.	W	− 4
12	+15	+15	+14	+12	+11	+ 7	+11	+15	+20	+22	+20	+16	1910 July and Aug.	S	+15
13	− 6	− 8	− 9	− 7	− 6	− 6	− 6	− 5	− 3	− 1	0	− 2	,, Oct. to Dec.	W	− 5
14	+ 3	− 4	−12	−11	− 6	− 2	+ 2	+ 5	+ 7	+ 7	+ 6	+ 5	1911 June and July	S	0
15	+ 3	− 8	−16	−18	−18	− 8	+ 4	+10	+14	+14	+13	+ 9	1912 June and July	S	0
16	+ 2	0	− 2	− 4	− 7	0	+ 6	+ 6	+ 6	+ 7	+ 7	+ 5	,, Oct. and Nov.	W	+ 2
17	− 5	− 6	− 8	−10	− 9	− 7	− 5	− 1	+ 6	+10	+ 5	− 5	1913 June and July	S	− 3
18	− 5	− 8	− 7	− 7	− 6	− 6	− 4	− 1	+ 2	+ 3	+ 1	− 2	,, Sept. to Nov.	W	− 3
19	+ 1	− 8	−17	−20	−20	− 9	+ 3	+10	+16	+16	+14	+ 8	1914 July and Aug.	S	−0·5
20	+ 3	− 5	−13	−14	−12	− 4	+ 3	+ 5	+ 6	+ 8	+10	+ 8	1915 July and Aug.	S	0
23	+ 3	− 2	− 4	− 6	− 5	− 3	+ 2	+ 5	+ 6	+ 6	+ 6	+ 6	1918 July to Sept.	SW	(+1)
25	− 6	− 8	−15	−21	−15	−10	− 6	− 5	+ 8	+17	+ 9	+ 4	1920 Aug. and Sept.	W	−5
Mean	0	− 4	− 8	− 9	− 8	− 4	+ 1	+ 4	+ 7	+ 9	+ 8	+ 4	..	..	0
															S–W
															+5
							(b) Examples with Max. and Min. nearly midway between 2 points.								
1	− 4	− 4	− 4	− 3	− 3	− 2	− 1	+ 6	+ 9	+ 8	+ 6	+ 0	1906 May and June	S	+1
2	− 1	− 4	− 7	− 7	− 5	− 1	+ 3	+ 5	+ 7	+ 5	+ 3	+ 1	,, Aug. and Sept.	W	0
3	− 3	− 4	− 5	− 6	− 6	− 6	− 1	+ 3	+ 5	+ 5	+ 1	− 3	,, Oct. to Dec.	W	−2
4	+ 1	− 3	− 7	− 8	− 5	− 2	+ 4	+ 8	+12	+14	+10	+ 5	1907 Mar. to May	S	+2
5	+ 4	+ 3	+ 2	+ 1	+ 1	+ 6	+ 8	+ 8	+ 8	+ 9	+ 9	+ 7	,, June and July	S	+5·5
6	− 2	− 4	− 6	− 7	− 4	− 1	+ 3	+ 6	+ 7	+ 7	+ 5	+ 1	,, Sept. and Oct.	W	0
9	0	− 2	− 3	− 2	0	+ 2	+ 3	+ 7	+10	+ 8	+ 7	+ 3	1909 May and June	S	+3
10	0	− 4	− 5	− 5	− 4	− 3	+ 5	+ 8	+ 9	+ 9	+ 6	+ 3	,, July and Aug.	S	+2
11	− 3	−10	−15	−14	−10	− 2	+ 6	+ 9	+11	+10	+ 7	+ 2	,, Nov. and Dec.	W	−1
21	−10	−14	−17	−16	−14	−11	− 7	+ 2	+ 9	+ 7	+ 2	− 4	1916 Oct. and Nov.	W	−6
22	− 2	− 5	− 8	− 9	− 7	− 5	0	+ 5	+ 7	+ 7	+ 4	+ 1	1917 Aug. to Nov.	W	−1
24	− 6	−15	−23	−19	−12	0	+ 4	+ 3	+ 2	0	− 2	− 4	1919 Sept. and Oct.	W	−6
Mean	− 2	− 6	− 8	− 8	− 6	− 2	+ 2	+ 6	+ 8	+ 7	+ 5	+ 1	..	..	0
															S–W
															+5

The first column in this table gives the number of the example (corresponding to that in Table IV.); the next 12 give the figures of the 12 equidistant points; the next the date; the next "S" or "W" (see Table III.); and the final column the mean residual in each example. The value of "S−W," given at the foot of each division, agrees in both cases with its mean value in Table III. Each alternate figure in the columns (1) to (12) is of course the same as in Table IV. The remarkable general faintness in No. 12 is equally shown by both observers. Only 5 negative residuals occur, 2 by Ma and 3 by Mo, 3 of them on two consecutive nights, none of them large, and all within portions 5, 6, and 1 in Table IV. But negative residuals begin to preponderate immediately after the end of this group, which ends on August 14.

As it may be of interest to compare these results with those in the Baxendell paper, the latter are given below:

	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).	(10).	(11).	(12).	No. of Examples.
a	0	−3	−6	−8	−6	−3	0	+3	+6	+8	+6	+2	14
b	−2	−5	−7	−7	−4	−1	+1	+3	+7	+7	+5	+2	7

It will be seen that almost the only difference is in amplitude. This close accordance is as satisfactory as it is surprising, considering all the conditions, and I cannot help feeling that it removes any possible doubts as to the genuineness of this secondary variation, whatever its cause may be.

The amplitude comes out (by harmonic analysis) at between 8 and 9 (0·08 and 0·09 magnitude) in both divisions. In the Baxendell sets it was over 7. The form is a still nearer approach to a sine curve than in the previous work.

In the groups of cycles the residuals were used without any alteration except the personal correction, and interpolations were sparingly and cautiously introduced. There was no smoothing, except that, in No. 1, the observations being awkwardly distributed, the means of adjacent pairs were used in addition to the observations as they stood. This was not done in any other case.

The details of No. 19 are given as an example. The lines *a*, *b*, etc., are for consecutive cycles of 6·57 days. The last residual (+13) comes so close to the boundary between (5) and (6) that it is given in both. The −2 in line *f* is the only interpolation, and it seems to be reasonably justified. A gap follows the line *h*.

	(1).	(2).	(3).	(4).	(5).	(6).
a	..	..	−31	+7	+10	+11
b	+13	− 1	..	..	..	..
c	− 8	−36	−7	..	+24	..
d	− 1	..	..	..	..	..
e	..	..	..	..	..	+10
f	− 8	−14	−22	[−2]	+18	+21
g	..	..	..	+4	+17	+16
h	+11	..	..	..	+11 +13	+13
Mean	+ 1	−17	−20	+3	+16	+14

One object in giving this illustration is to show how little importance can be attached to the *precise* phase deduced for examples so sparsely filled as this one is. By starting one cycle later, by stopping one cycle earlier, or by omitting the interpolated figure, a difference in phase of a few degrees might be got; but in any case (2) and (3) would still have the highest negative, and (5) and (6) the highest positive values, so that the phase would never be very far from 270°. A few observations by Ma were available in the earlier part of this group, but, as several of these are doubtful and there are none in the latter part, it seemed better to keep this example to Mo only.

The object of this investigation having thus been attained, no attempt has been made to find any other real or apparent systematic variations, and none have shown themselves in the course of the work.

In regard to the irregularities to which Colonel Markwick calls attention in the black-dot light-curve in the *B.A.A.J.* (see above), it seems possible that they may be at least partly due to the following circumstance. In the observed cycles the maximum of the secondary variation occurs more frequently within $1\frac{1}{2}$ days of the minima of β Lyræ's primary variation than it does near the latter's maxima. This is not, so far as I can judge, through any tendency towards that position, but through the gaps in the observations happening to favour such a result. Though the details have not been fully worked out, this has been ascertained to be the case, not to any very great extent, but probably sufficiently to account for the dip in the black dots at the maxima, and for what is real in the rises near the minima.

Colonel Markwick has very kindly offered to lend me the observations from which his open-dot curve is formed, which were taken in the years 1899 to 1905; an offer which I most gratefully accept. With those observations in hand, I hope to make an attempt at collating the whole of the three series. It is very important to discover the apparent mean period of this variation between the dates already dealt with, and to ascertain whether the fluctuation in phase appears again, and if so to determine its probable period more accurately.

In order to judge whether the phases of β Lyræ's primary variation produce any effect on this secondary one, the difference between each individual value in the columns (1) to (12) of Table V. and the mean at the foot of its column was taken, and the resulting figures were tabulated in shifted positions to accord as nearly as possible in each case with the phases of the primary variation. The accordances could not be exact, as the assumed (B) period was 0·11 days longer than half the primary cycle. Also, of course, the 2 minima of the primary one come together here and so do the 2 maxima, so that a differential effect between either of these would not appear. (There is, however, no sign of any such systematic difference in the alternate cycles of (B).)

A summary of the figures thus tabulated is given below, the unit here being 1 magnitude.

TABLE VI.

Phases of the Primary Variation.

No. of Observers.	No. of Examples.	Min.	Rise.	Max.	Fall.	Total.
3 and 4	6	0·000	−0·011	+0·008	+0·008	+0·001
2	6	+0·030	+0·028	+0·039	+0·022	+0·030
1 (Ma and Ry)	6	−0·014	−0·001	−0·011	−0·018	−0·011
1 (Mo)	7	−0·022	+0·003	−0·023	−0·021	−0·016
Total	25	−0·002	+0·005	+0·002	−0·003	+0·0004
Bax.	22	+0·008	−0·003	−0·007	0·000	−0·0004

The figures in each of the phase columns are from three consecutive points (combined) in Table V. Despite the appearance of system, the results are small enough to be accidental, especially as no two of the separate divisions agree in form. The Baxendell results, given for comparison, reverse the results pretty nearly.* If these figures are not accidental they might represent very small errors in the form of the curve or in the epoch formula, or some very small long-period variation.

In the course of this work most of the observations were gone through for the residuals six times.

1. With the Baxendell curve and B.A.A. formula.
2. With first corrected ,, ,, ,,
3. With final form of ,, ,, ,,
4. With final curve and first corrected formula.
5. With ,, and second ,, ,,
6. With ,, and final ,, ,, (as given above).

Also, the residuals were corrected for personality several times over, in the course of forming and testing the details of Table II.

Summary.

1. A series of 1500 observations of β Lyræ, taken by four members of the B.A.A., extending over 16 years and representing more than 400 periods, have been collated, and corrections for personal differences have been deduced and applied (Table II.).

2. A revised formula (for date of principal minimum)

$$2398590{\cdot}57+12{\cdot}908006E+0{\cdot}000003914E^2$$

was deduced from them, with a light curve shown in Table I., differing but little from that of Baxendell.

3. A small seasonal effect was found (Table III.), similar to that shown in Baxendell's observations, but differing from it in some respects.

4. The subsidiary variation (B), found in Baxendell's observations, appears again in these, with an apparent mean period of about 6·595 days, or 0·019 days longer than before. A fluctuation in its phases similar to the one previously found is also shown, but the magnitude of the swing now appears to be 1·48 days on either side of the mean

* These were not given in the former paper.

value in a period of 3311·28 days, instead of 1·22 days in 3562·608 days, as previously found.

5. The existence of this subsidiary variation has thus been fully confirmed, and the apparent slight lengthening of its mean period is probably real. The evidence for a long-period fluctuation in its phase is strengthened, but the precise amount and character of this remain doubtful.

Note.—I have again to thank Professor Turner for allowing me to send him a draft of this paper. He also very kindly sent a copy of the Baxendell paper (frequently referred to above) to Dr. Rossiter of Michigan, U.S.A. (who has been studying β Lyræ's variations spectroscopically), asking whether his investigations showed any sign of this secondary variation. The reply was in the negative, which seems to preclude any likelihood of its being due to a third revolving body. But Dr. Rossiter permits us to mention that he has found an oscillation of the line of apsides in about a 9-years period (not determined nearer than a large fraction of a year), and it seems possible that this may have some connection with the fluctuation period of between 9 and 10 years found in the secondary variation investigated above.—M. A. B.

A New Variable Star in Perseus. By A. Stanley Williams.

B.D.+36° 736 (9·1 mag.).

R.A.=3 h 33 m 01 s, Decl.= +36° 02′·0 (1855).
R.A.=3 35 55, Decl.= +36 10·9 (1900).

The variability of the above star does not seem to have been noticed hitherto. It was first recognised to be variable by the writer in January last, when comparing some photographs taken over twenty years ago with a 4·4-inch portrait lens. Visual observations were commenced at the first opportunity with a 6½-inch reflector. The following comparison stars have been used. A=B.D. 36° 739 (8·7 mag.), a=B.D. 35° 729 (9·3 mag.), b=B.D. 35° 735 (9·4 mag.), and a star c which lies about 5′ n.f. the variable. Estimations of the brightness of the latter on the photographs are given below. The value of a step is approximately 0·05 mag.

Date.	G.M.T. h m	Photographic Observations.	
1901 Feb. 11	11 30	c6v	poor.
Apr. 14	8 50	c1v	,,
Dec. 18	15 02	c5v	,,
1902 Mar. 5	10 25	c3v	very good.
Dec. 23	15 25	v20±c, a3v	good.
,, 29	14 46	v20±c, a1v	,,
1903 Jan. 2	14 38	v20±c, a1v	very good.
Mar. 3	10 45	v5c	poor.
1904 Jan. 19	13 39	v5±c	very poor.
Feb. 15	11 52	v1c	very good.

The times are the middle times of the exposures. Nothing very definite is indicated as to the nature of the variability, but the observed photographic range of variation amounts to nearly a magnitude and a half. The visual observations follow below.

Date.	G.M.T.* h m	Visual Observations.	Wt.
1925 Jan. 26	22 07	v2a, v10b, v20±c	2
27	20 31	v2a, v9b	3
28	18 37	v5a, v12b	5
30	23 09	v7a, v14±b	2
31	18 12	v6a, v12b	3
Feb. 2	19 36	v7a, v14b	5
6	21 53	A2v, v8a	5
9	20 00	A4v, v6a	4
13	20 00	A7v, v1a, v10b	5
14	20 55	A6v, v2a, v10b	3
17	19 48	A4v, v3a, haze ?	1
18	19 33	A7v, v1a, v13b	5
19	19 56	A8v, v1a, v10b	2
23	19 44	A9v, a2v, v10b	5
27	19 27	A10v, a2v, v9b	5
Mar. 2	19 42	A10v, a1v, v10b	3

Only one observation is given for each night, but on several nights the star was observed at intervals during two or three hours. In no case, however, was any perceptible change perceived on any one night. A very short period is, therefore, unlikely. The value of one step is probably about 0·05 mag.

The visual observations indicate a maximum about 1925 February 6, but the curve is very flat, so that the exact date is rather uncertain. Having regard to both the visual and the photographic observations, it is probable that the variable is one of long or moderately long period, or else an irregular variable. The star is not highly coloured.

Polvarth, St. Mawes :
1925 *March* 2.

Note on the Proper Motion of the Variable Star, Harvard 2435, *in the Great Magellanic Cloud.* By H. Spencer Jones, H.M. Astronomer.

The variability of the star Harvard 2435, in the Great Magellanic Cloud, was discovered by Miss Leavitt (*Harvard Annals*, vol. **60**, No. 4). The limits of magnitude have been determined at Harvard as 10·8 m. and 11·7 m. (photographic) and its spectral type found to be K5 (*Harvard Bulletin*, No. 754). Professor Hertzsprung, at Johannesburg, has recently found that the variable is of the W Ursæ Majoris type, the half period being $0^{d}\cdot14273$, with a minimum at 2423784·600 J.D., M.G.T. Its position is $5^{h}\ 18^{m}\ 59^{s}\cdot9$, $-68^{\circ}\ 21'\cdot1$ (1875).

* From midnight.

Professor Hertzsprung pointed out to me that owing to its great density the star should be a dwarf and have a sensible proper motion. From its spectral type a visual magnitude of about 2 m. may be expected, when the star is placed at a distance of one parsec. The observed visual magnitude of about 10 m. should thus correspond to a parallax of about ″·025. A velocity of 20 km. per sec. at this distance, which is of the order of magnitude to be expected for a K5 star, would correspond to a proper motion of about one-tenth of a second of arc per year.

The following plates of the region, taken with the astrographic telescope, are available in the Cape series :

No. 4726	1898 January 21	Exposure 4 hours.
,, 4730	,, ,, 25	,, 4 ,,
,, 24	1917 October 25	,, 44½ minutes.

Two further plates were exposed through the glass, to enable the proper motion to be derived by direct comparison with the earlier plates :

No. 1504	1925 January 15	Exposure 15 minutes.
,, 1507	,, ,, 18	,, 15 ,,

A new and an old plate, placed film to film, were compared, measurements of the variable and of about 12 to 14 symmetrically placed adjacent stars being made. The following are the results of the comparisons :

	Interval.	Δx.	Δy.
	y	r	r
4730–1507	26·981	+0·105	+0·041
4730–1504	26·975	+ ·096	+ ·038
4726–1504	26·986	+ ·106	+ ·035
24–1504	7·228	+ ·023	+ ·008
24–1507	7·234	+ ·026	+ ·008

The derived values of the centennial proper motion with the assigned weights are :

			Weights.
4730–1507	+11″·63	+4″·55	6
4730–1504	10·64	4·12	6
4726–1504	11·76	3·89	3
24–1504	9·51	3·22	0·5
24–1507	10·73	3·22	0·5

Plate 4726 has double images ; only comparison stars of about the same magnitude as the variable were used. It has been given half the weight of 4730.

The concluded value of the centennial proper motion is

$$\mu_\alpha = +11''\cdot2, \qquad \mu_\delta = +4''\cdot2,$$

which is of the order of magnitude anticipated.

The parallactic motion of the star, corresponding to a parallax of 0″·025 and a solar velocity of 19·5 km. per sec., would be 5″·9 per century or +1″·5 in right ascension and +5″·7 in declination. The proper motion is therefore mainly not of parallactic origin.

The magnitude of the proper motion indicates that the variable is not physically connected with the Magellanic Cloud. The direct determination of the star's parallax is to be desired.

The Density-function in the Milky Way. By Dr. E. A. Kreiken.

§ 1. In the following we put:

m the apparent magnitude of a star;
M its absolute magnitude related to the apparent by the formula

$$M=m-\rho \text{ where } \rho=5\log r;$$

r its distance from the sun (unit = 1 parsec);
β and λ its galactic latitude and longitude;
$|\beta|$ the absolute value of its galactic latitude;
$A(m)$ the number of stars per square degree between magnitudes $m-\frac{1}{2}$ and $m+\frac{1}{2}$;
$N(m)$ the number of stars per square degree from the brightest to magnitude m;
$D(\rho)=D(r)$ is the Density-function;
$\phi(M)$ the Luminosity-function.

When giving the formulæ of $A(m)$, $N(m)$, $D(\rho)$, and $\phi(M)$ as Dr. A. Pannekoek * has used them, and also adopting his modification of the Schwarzschild formulæ, we have:

$$\log A(m)=a+b(m-m_0)+c(m-m_0)^2 \quad . \quad . \quad . \quad (1)$$

$$\log \phi(m)=p+qM-rM^2 \dagger \quad . \quad . \quad . \quad . \quad (2)$$
$$(p=-2\cdot394\,;\ q=+0\cdot186\,;\ r=+0\cdot0345).$$

$$\log D(\rho)=h+k(\rho-\rho_0)-l(\rho-\rho_0)^2 \quad . \quad . \quad . \quad (3)$$

$$\left.\begin{aligned} &\rho_0=m_0+\frac{b-0\cdot186}{0\cdot069} \ . \ . \ . \ (a)\,;\ \frac{1}{l}=\frac{1}{c}-29 \ . \ . \ . \ (b)\,;\ k=b-0\cdot6 \ . \ . \ . \ (c)\\ &h=a+4\cdot937-0\cdot6m_0+\frac{k^2}{0\cdot138}+\tfrac{1}{2}\log(l+0\cdot0345) \ . \ . \ . \ (d)\end{aligned}\right\} \quad (4)$$

Counting the numbers of stars $A(m)$ between the limits of apparent magnitude 10·00–10·99; 11·00–11·99, etc. in the areas in the "Durch-

* Dr. A. Pannekoek, *Proc. Amsterdam Acad.*, **24**, 1921. Cf. also *Publ. Amsterdam Astr. Laboratory*, No. 1, p. 7.

† J. C. Kapteyn and van Rhijn, "On the Distribution of Stars," *Astroph. Journ.*, **52**, 289.

musterung of the Selected Areas " * of low galactic latitude, I found a marked systematic difference between the apparent luminosity curves in the different areas.

These differences are to a large extent to be ascribed to the fact that the apparent luminosity curve is different for the different galactic longitudes. There remains, however, a difference between the results in areas with nearly equal galactic longitudes.

The dependency of the apparent luminosity curve on longitude appears from Tables I. and II. In the first table the mean values log N_m are given for different longitudes and for $|\beta| < 10°$. Table II. contains the corresponding values for $|\beta|$ from 10° to 20°. These values were derived by combining all areas with λ from 0°–60° into one set;

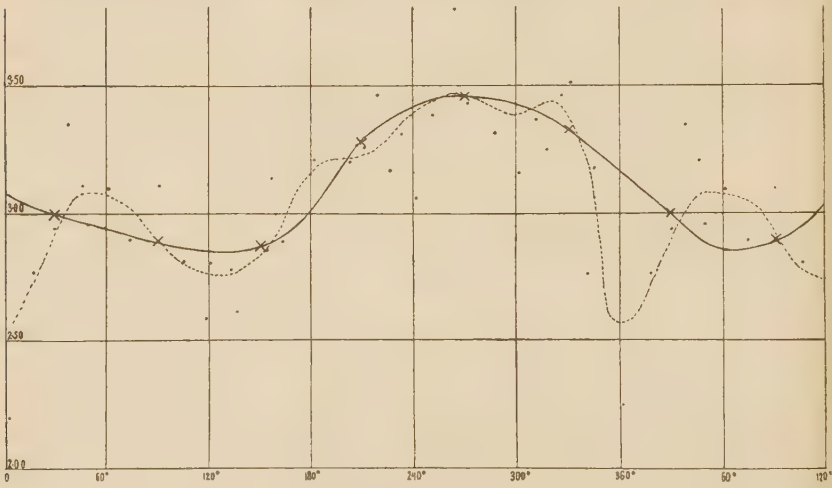


FIG. 1.

The values of log N 14·99 on the left side of Table I. are indicated by crosses. The dots indicate the values of log N 14·99 for the separate areas.

the same was done with the areas with λ from 60°–120°, etc. For each set the total number of stars up to magnitude m were counted. From the totals we find the values log N_m, *i.e.* the logarithm of the total number per square degree.

On the right side of Tables I. and II. I give the difference between the value of log N_m for each longitude and the value for λ120°–180°.

In Tables I. and II. the star numbers show a minimum for λ120°–180° and a maximum for λ240°–300°.

To what extent these mean values represent the actual distribution of the star numbers appears from figs. 1 and 2, where the values of log $N_{14.99}$ in Tables I. and II. have been indicated by crosses. Through the points indicated by those crosses a smooth curve has been drawn (in the figures the full-drawn line).

* E. C. Pickering and J. C. Kapteyn, "Durchmusterung of Selected Areas," *Harvard Annals*, ci. The results for the southern areas were kindly placed at my disposal in advance of publication by Prof. P. J. van Rhijn.

The values of log N_m for each separate area are indicated by a dot. Through these dots a second curve was drawn (dotted line). Except for λ about 0° the agreement between the two curves is satisfactory. At λ about 0° the dotted curve has a sharp minimum which disappeared in the curve of the mean values.* We will therefore give a separate treatment for this region. The differences between the apparent luminosity curves in closely adjacent regions appear in Table III.

The value log N_m rich for λ from 0°-60° has been found by dividing the areas with λ from 0° to 60° into two groups, the first containing the richer areas, the second the poorer areas. In the same way two groups were formed from the areas with λ from 0° to 60° and $|\beta|$ from 10° to 20°. The richest group of areas with $|\beta| < 10°$ were combined into one set with the richest group with $|\beta|$ from 10° to 20°. A second set was

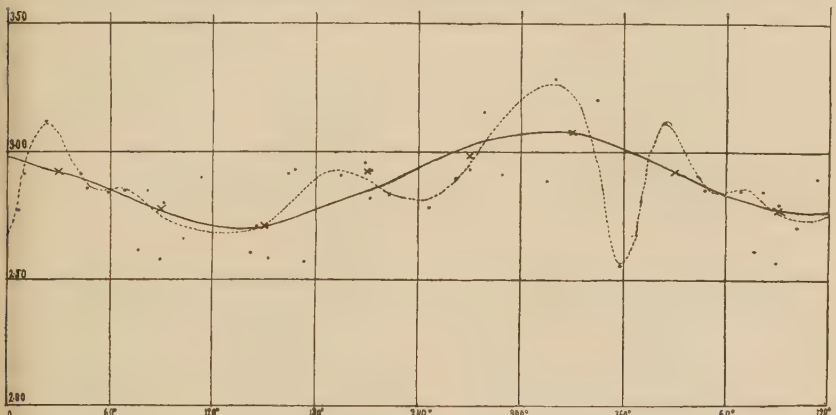


FIG. 2.

The values of log N 14·99 on the left side of Table II. are indicated by crosses. The dots indicate the values of log N 14·99 for each separate area.

formed from the remaining poorer areas. From the first set we find the value log N_m rich, and from the second the value log N_m poor. For the other longitudes the corresponding values have been found in the same way. Besides these values, Table III. also contains the differences log N_m rich−log N_m poor. From these differences it appears that for each galactic longitude there remains a systematic difference between the apparent luminosity curves of the areas they contain.

As it is evident that the differences in the apparent luminosity curves are due to a difference in the space distribution of stars, it is also evident that we will have

1. A different density-function for the different longitudes (Tables I. and II.).

2. A different density-function for the richer and poorer areas in each longitude (Table III.).

In the following we will therefore try to derive, firstly, a density-

* No difference was found between the areas with β from −10° to −20° and +10° to +20°.

Table I.

$|\beta| < 10°$. *log* N_m *for each Longitude and* $\Delta(\log N_m - \log N_m\ 120°\text{–}180°)$.

Gal. Long. / Photg. Magn.	log N_m. 0°–60°.	60°–120°.	120°–180°.	180°–240°.	240°–300°.	300°–360°.	Δ. 0°–60°.	60°–120°.	120°–180°.	180°–240°.	240°–300°.	300°–360°.
9·99	1·142	1·036	0·995	1·207	1·282	1·246	+·147	+·041	·000	+·212	+·287	+·251
10·99	1·575	1·425	1·266	1·685	1·738	1·565	·309	·159	·000	·419	·472	·299
11·99	1·987	1·836	1·753	2·176	2·240	2·026	·234	·083	·000	·423	·487	·273
12·99	2·368	2·217	2·174	2·556	2·692	2·525	·194	·043	·000	·382	·518	·351
13·99	2·660	2·573	2·511	2·962	3·082	2·931	·149	·062	·000	·451	·571	·420
14·99	3·010	2·896	2·886	3·274	3·463	3·319	·124	·010	·000	·388	·577	·433

Table II.

$|\beta|$ *from* 10° *to* 20°. *log* N_m *for each Longitude and* $\Delta(\log N_m - \log N_m\ 120°\text{–}180°)$.

Gal. Long. / Photg. Magn.	log N_m. 0°–60°.	60°–120°.	120°–180°.	180°–240°.	240°–300°.	300°–360°.	Δ. 0°–60°.	60°–120°.	120°–180°.	180°–240°.	240°–300°.	300°–360°.
9·99	0·971	0·954	0·975	0·915	0·955	0·954	−·004	−·021	·000	−·060	−·020	−·021
10·99	1·442	1·275	1·267	1·163	1·287	1·330	+·175	+·008	·000	−·104	+·020	+·063
11·99	1·840	1·723	1·667	1·726	1·830	1·759	·173	·056	·000	+·059	·163	·092
12·99	2·204	2·097	2·002	2·204	2·271	2·230	·202	·095	·000	·202	·269	·228
13·99	2·520	2·408	2·340	2·505	2·645	2·602	·180	·068	·000	·165	·305	·262
14·99	2·913	2·776	2·713	2·919	2·976	3·066	·200	·063	·000	·206	·266	·353

Table III.

Difference between log N_m Richer and Poorer Region.

Gal. Long. / Photg. Magn.	log N_m rich.						log N_m poor.						Δ (log N_m rich–log N_m poor).					
	0.	60.	120.	180.	240.	300.	360. 0.	60.	120.	180.	240.	300.	360. 0.	60.	120.	180.	240.	300.	360.
9·99	1·544	1·462	1·447	1·580	1·681	1·556	1·431	1·505	1·230	1·431	1·301	1·431	+·113	−·043	+·217	+·149	+·380	+·125
10·99	2·045	1·914	1·699	1·949	2·083	1·881	1·820	1·748	1·556	1·949	1·756	1·763	·225	+·166	·143	·000	·327	·118
11·99	2·446	2·358	2·143	2·520	2·601	2·342	2·217	2·155	1·973	2·338	2·267	2·201	·229	·203	·170	·182	·334	·141
12·99	2·833	2·743	2·555	2·901	3·063	2·855	2·572	2·521	2·362	2·771	2·694	2·676	·261	·222	·193	·130	·369	·179
13·99	3·144	3·102	2·931	3·287	3·460	3·279	2·852	2·809	2·667	3·131	3·067	3·029	·292	·293	·264	·156	·393	·250
14·99	3·523	3·444	3·309	3·628	3·834	3·732	3·199	3·161	3·040	3·476	3·419	3·342	·324	·283	·269	·152	·415	·390

Table IV.

Values of the Coefficients.

Gal. Long. / Photg. Magn.	$\lvert\beta\rvert < 10°$.							$\lvert\beta\rvert$ from 10°–20°.						
	a.	β.	a.	b.	ρ_0.	h.	k.	a.	β.	a.	b.	ρ_0.	h.	k.
0–60	−2·533	+·374	+2·485	+·394	16·0	− ·802	−·206	−2·868	+·388	+2·360	+·409	16·2	− ·970	−·191
60–120	2·653	·372	2·337	·392	16·0	·956	·208	2·678	·364	2·195	·384	15·9	1·061	·216
120–180	2·894	·386	2·305	·407	16·2	1·019	·193	2·506	·347	2·115	·366	15·6	1·082	·234
180–240	2·865	·413	2·724	·435	16·6	·673	·165	3·114	·401	2·305	·423	16·4	1·062	·177
240–300	3·108	·443	2·924	·467	17·1	·542	·133	3·056	·405	2·421	·427	16·5	·956	·173
300–360	2·975	·418	2·696	·440	16·7	·712	·160	3·285	·422	2·450	·444	16·7	·976	·156
−25° to +25°	2·405	·352	2·291	·371	15·8	·923	·229	3·075	·400	2·329	·422	16·4	1·035	·178

function for each longitude; and secondly, the ratio between the space density at the same distance from the sun in the direction of the richest and of the poorest areas.

We may derive the values of the coefficients h, k, and l in the density-function from formula (4) if the coefficients a, b, and c of the apparent luminosity curve are known. As the range of our magnitudes is rather small, it will not be possible to find an accurate value for c.

Therefore the numbers $\log N_m$ in Tables I. and II. were represented by linear * formulæ

$$\log N_m = \alpha + \beta m \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It appears that the deviations from these linear formulæ are rather small. If in (5) we write visual instead of photographic magnitudes, we have

$$\log N_m = \alpha' + \beta' m \quad . \quad . \quad . \quad . \quad (6)$$

From table 33 in *Gron. Publ.*, 27, I find

$$m_{ph} - m_{vis} = +0{\cdot}52 + 0{\cdot}05(m_{ph} - 12) \quad . \quad . \quad . \quad (7)$$

From (5), (6), and (7) we find

$$\beta' = \frac{\beta}{0{\cdot}95}; \quad \alpha' = \alpha - 0{\cdot}8\beta'.$$

We have

$$A(m) = a' + b'm + c'm^2 = \frac{\partial N m}{\partial m},$$

so

$$c' = 0; \quad b' = \beta', \text{ and } a' = \alpha' - \log\log e + \log\beta'.$$

If we write

$$A(m) = a' + b'm = a + b(m - 13) + c(m - 13)^2,$$

we have

$$c = 0; \quad b = b', \text{ and } a = a' + 13b'.$$

From a, b, and c and from formulæ (4) we find the constants h, k, and ρ_0 of the density-function ($l = 0$).

For each longitude the values of h, k, ρ_0, α, β, a, and b are given both for $|\beta| < 10°$ and $|\beta|$ from 10° to 20° in Table IV.

From the function

$$\log D(\rho) = h + k(\rho - \rho_0) \quad . \quad . \quad . \quad . \quad (8)$$

we now find the space density at different distances from the sun.

Table V. contains the results for different longitudes.

We may only expect reliable results from the density-function $D(\rho)$ between the limits where it is determined by the observed luminosity curve. We find these limits by writing in 4(*a*) instead of m_0 the values $m_{vis} = 14{\cdot}3$ and $m_{vis} = 9{\cdot}6$. In Table V. the values outside these limits are printed in italics.

* Pannekoek has shown (*loc. cit.*) that the value of the function $D(\rho)$ at the distance $\rho_0 = m_0 + \frac{b - 0{\cdot}186}{0{\cdot}069}$ will be chiefly determined by the value of the coefficient a. A change in b will have but small influence, and a change of c no influence at all.

A survey of the results obtained in Table V. is given in fig. 3, in which the lines of equal density in the zone $|\beta| < 10°$ have been drawn. I

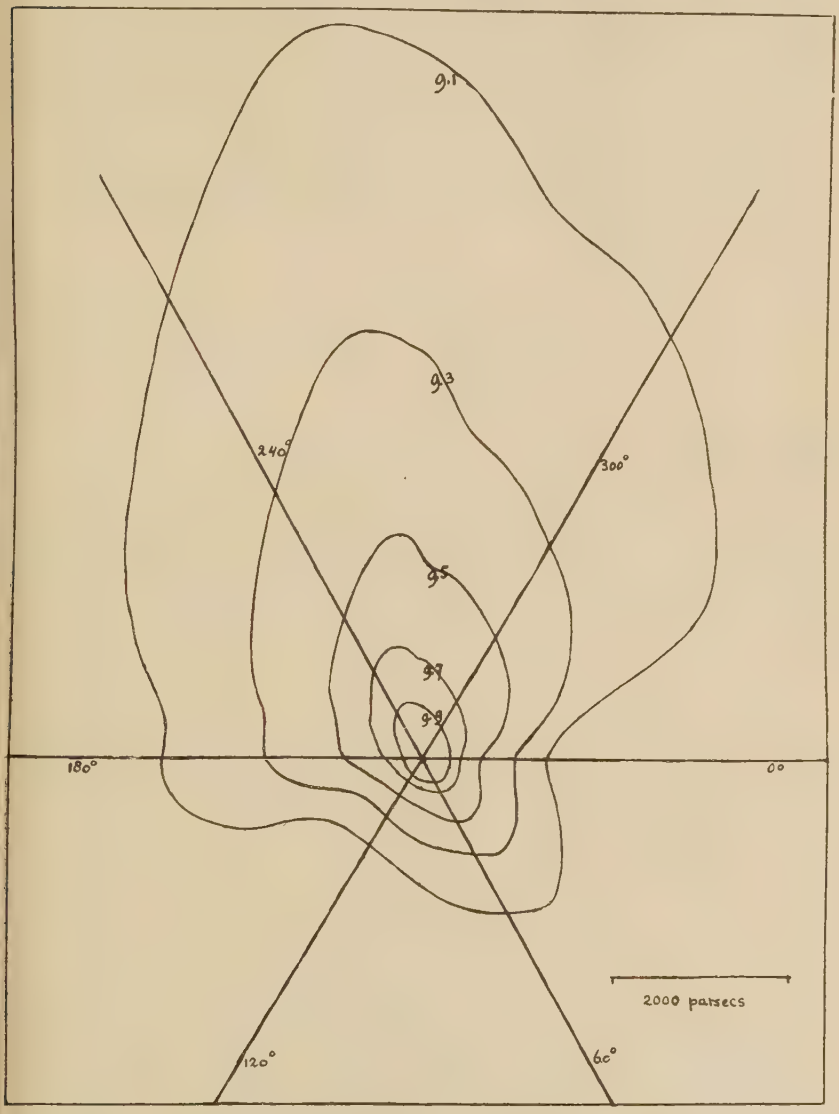


FIG. 3.—Curves of equal Density, $1\beta/<10°$.

have given no separate figure for $|\beta|$ 10° to 20°, as this figure proved to be closely similar to fig. 3.

From this figure it appears that the star system is chiefly elongated

in the direction of λ about 270°. Between λ30° and 60° a second elongation of the system is to be found, though rather less conspicuous. This second elongation was already previously remarked by Dr. A. Pannekoek (*loc. cit.*). On the other hand, a scarcity of stars is to be found in the direction of Ophiuchus, between λ−25° and +25°. This scarcity was already indicated in the lines of equal density which Pannekoek derived from the brighter stars. It may be caused by dark obscuring matter, which screens the light of the stars behind it.

If any conclusion may be drawn at all from these curves about the situation of the centre of the star system, we should say it is to be found in the direction of λ about 270°. This result was previously suspected by Shapley.

TABLE V.

Density at Different Distances from the Sun. The unit is the Density in the Neighbourhood of the Sun.

	D(ρ) for |β|<10°.							D(ρ) for |β| from 10°–20°.						
ρ / Gal. Long.	12.	13.	14.	15.	16.	17.	18.	12.	13.	14.	15.	16.	17.	18
0°–60°	*0·02*	9·82	9·61	9·40	9·20	8·99	*8·79*	*9·83*	9·64	9·45	9·27	9·07	8·87	8·
60°–120°	*9·88*	9·67	9·46	9·25	9·04	8·84	*8·63*	*9·79*	9·57	9·35	9·14	8·92	8·71	8·
120°–180°	*9·79*	9·60	9·41	9·21	9·02	8·83	*8·63*	*9·76*	9·52	9·29	9·05	8·82	8·59	8·
180°–240°	*0·08*	9·92	9·75	9·59	9·42	9·26	*9·09*	*9·72*	9·55	9·37	9·19	9·02	8·84	8·
240°–300°	*0·14*	*0·01*	9·87	9·74	9·61	9·47	*9·34*	*9·82*	9·65	9·47	9·30	9·13	8·95	8·
300°–360°	*0·04*	9·88	9·72	9·56	9·40	9·24	*9·08*	*9·76*	9·60	9·45	9·29	9·13	8·98	8·
−25° to +25°	*9·95*	9·72	9·49	9·26	9·03	8·80	*8·58*	*9·75*	9·57	9·39	9·21	9·04	8·86	8·

TABLE VI.

Values of h_1–h_2, k_1–k_2, and ρ_0 for Different Longitudes.

λ.	0°.	60°.	120°.	180°.	240°.	300°.	360°.
h_1–h_2	+·327	+·383	+·225	+·131	+·370	+·425	
k_1–k_2	+·044	+·068	+·009	+·003	+·007	+·055	
ρ_0	15·7	15·7	15·4	16·0	16·3	16·3	

TABLE VII.

$\Delta(\log D_1(\rho)-\log D_2(\rho))$.

λ / ρ.	0°.	60°.	120°.	180°.	240°.	300°.	360°.
12	+·17	+·13	+·20	+·12	+·34	+·19	
13	·21	·20	·21	·12	·35	·24	
14	·26	·26	·22	·12	·35	·30	
15	·30	·33	·23	·13	·36	·35	
16	·34	·40	·24	·13	·37	·41	
17	·39	·47	·25	·13	·37	·46	

If $\log D_1(\rho)=h_1+k_1(\rho-\rho_0)+l_1(\rho-\rho_0)^2$ represents the density-function in the richest areas,

and $\log D_2(\rho)=h_2+k_2(\rho-\rho_0)+l_2(\rho-\rho_0)^2$ the density-function in the poorer areas,

then we have, if in both formulæ we adopt the same value of ρ_0,

$$\log \frac{D_1(\rho)}{D_2(\rho)}=(h_1-h_2)+(k_1-k_2)(\rho-\rho_0)+(l_1-l_2)(\rho-\rho_0)^2 \quad . \quad (9)$$

From formulæ (4) we find

$$k_1-k_2=b_1-b_2$$

$$h_1-h_2=a_1-a_2-0{\cdot}6(m_0 \text{ rich}-m_0 \text{ poor})+\frac{(b_1-b_2)(b_1+b_2-1{\cdot}2)}{0{\cdot}138},$$

while $l_1=l_2=0$.

For the rich areas we take $m_0=13$. Then 4(*a*) gives ρ_0, and the value (m_0 poor) is found from

$$m_0=\rho_0-\frac{b-0{\cdot}186}{0{\cdot}069}.$$

In Table VI. the values h_1-h_2, k_1-k_2, and ρ_0 have been entered. Table VII. gives the ratio of the density at the same distance as it is found from formula (9).

It appears that in the direction of greatest elongation the change of the ratio with increasing distance is small. In the other direction there is a rapid change. It is of very great importance to note that a change in the ratio of the space density in closely adjacent regions may already occur within $\rho=12$ or $r=250$ parsecs.

Summary.

1. The apparent luminosity curve is systematically dependent on the galactic longitude.

2. The curves of equal density show that our star system is chiefly elongated in the direction λ about 270°.

3. Within a distance of $r=250$ we may expect in closely adjacent regions a ratio of 2 : 1 in the space density.

Haarlem, Holland.

Note on Errors in Time-Determination as shown by Comparison of W/T Time-Signals. By H. Knox-Shaw.

The object of this note is to point out the possibility not only of discovering errors in the determination of time at any particular observatory by comparing the observations of W/T time-signals made at a number of observatories, for this has already been done by Professor Sampson,* but also in some cases of determining the actual source of the error in the transit-circle observations.

* *Monthly Notices*, **82**, 215.

In *Helwân Observatory Bulletin*, No. 31, will be found an analysis of the Helwân observations with this end in view. All the observations available at the time of writing, from Paris, Greenwich, Edinburgh, Uccle, Helwân, Ottawa, and Washington, were employed to find the times of the "Mean Station" for the period 1922 November to 1924 February. The variations in the differences from the mean shown by each station are of such an order, the average standard deviation of a difference being $0^s{\cdot}05$, that it would clearly be desirable to have a larger number of stations on which to base the mean. This is the first point that I should like to emphasise. A satisfactory mean-station value could, I think, be obtained if observations were available from about a dozen stations. All that is necessary is that each station should observe its local time as accurately as possible, and one of the W/T time-signals daily. As a time-signal can be observed with a probable error so much smaller than that of a determination of time with the transit circle, it is not necessary for a co-operating observatory to observe more than one of the several series of rhythmic signals that are transmitted daily, though of course it is important that all signals employed should be observed at some station by a method that is free from personal equation.

The details of the analysis of the differences H−M (Helwân−Mean Station) are given in the paper already quoted, and only some of the results will be mentioned here. The values of H−M were listed for the days on which transits had been taken, and the changes in the values from one transit-day to the next were formed, these latter quantities being called X. Also, as from certain erratic variations in the observed values of the level constant of the transit circle it was suspected that there were errors involved in them, the values of $b \sec \phi$ were listed, and their differences, called Z, were formed in a similar manner. All those days were then chosen for which H−M exceeded $0^s{\cdot}05$, and the values of X and Z for these days were correlated. The value of the correlation coefficient found was $-0{\cdot}51$. This correlation showed that the changes in H−M were due in part to errors in the observed values of the level constant. Further scrutiny, and test of the spirit-level with which the level constant of the Helwân transit circle is determined, showed that there was an error of 10 per cent. in the adopted value of one division of the level.

Corrected values of all the figures necessitated by this change in the value of a division of the level are being published from Helwân. The value of the correlation coefficient between X and Z is not greatly affected thereby, the revised value being $-0{\cdot}45$. Thus the Helwân times are still affected by errors in the observed values of the level constant. As pointed out in Bulletin No. 31, these may well be caused by particles of dust on the pivots of the transit circle. A sample of the dust was examined, and found to contain particles of a size amply sufficient to cause irregularities of the order found. Unless the dust difficulty can be obviated by taking more stringent precautions than has been done at Helwân in the past, this furnishes a strong argument against the use of the spirit-level, at any rate in a dusty climate, as a means of determining the level constant of a transit circle.

The second and main point of this note has been to draw attention to the fact that it has been possible by comparison of the observations of the W/T time-signals at several stations to show that at Helwân the errors in the time-determination are, at any rate in part, due to errors made in determining the level of the transit circle. If these could be got rid of, there would doubtless still be errors left, but the figures seem to show that they would be of a much smaller order than those which are due to errors in the determination of the level.

Radcliffe Observatory, Oxford:
1925 *February* 20.

Visibility of Stars in Great Britain during the Solar Eclipse of 1925 *January* 24. By J. K. Fotheringham, M.A., D.Litt.

I have to thank the Astronomer Royal for a public appeal for naked-eye observations of stars during the solar eclipse of 1925 January 24. From reports received I learn that throughout almost the whole of Great Britain observation of stars was rendered impossible by clouds. The only successful observations during the eclipse before sunset reported to me were by Mr. S. W. Hayes and Miss Hayes, at Primrose Cottage, Bucknell Road, Bicester, Oxfordshire, 51° 54′ N., 1° 10′ W., who saw Pollux at $16^h\ 15^m$ between clouds, and by Mr. A. S. Dow, at Darnaway, Morayshire, 57° 34′ N., 3° 41′ W., who enjoyed fine weather with haze round the horizon and saw Mars at $15^h\ 50^m$ and two stars in Cassiopeia, probably α and β, at $16^h\ 10^m$. I have made or confirmed the identifications by discussion with Mr. Hayes at the place of observation and by correspondence with Mr. Dow.

I have made the following computations :—

	Greenwich Mean Time.	Magnitude of Eclipse in Digits.	True Altitude of Sun.
	h m		°
Bicester, greatest phase . .	15 57	10·1	4·3
„ time of observation .	16 15	8·2	2·1
Darnaway, greatest phase . .	15 49	11·5	2·5
„ time of observation .	15 50	11·5	2·4
„ „ „ .	16 10	8·2	0·4

Refraction is ignored in the tabulated altitudes.

Similar observations of the eclipse of 1921 April 8 are printed in my *Historical Eclipses* (1921), pp. 26, 32. On that occasion there is no record of any star other than Venus being seen by observers who had not computed its exact position in advance, except where the eclipse attained a magnitude of at least 10·7 digits.* The sun's altitude at

* After the publication of my paper I learned from Mr. Pleydell that he had computed the exact positions of Vega and Altair in advance. I wish to modify accordingly the statement made on p. 32 of that paper.

the greatest phase of that eclipse at Oxford, where the magnitude was 10·7 digits, and where Venus, Vega, and Mercury were observed, was 29°. We may therefore attribute the success of the 1925 observers to the low altitude of the Sun, and must be cautious about the interpretation of records of the visibility of stars during an eclipse where the Sun was low.

It will be noticed that the stars seen were not the brightest in the sky. Vega and Capella were picked up at both places shortly after sunset.

University Observatory, Oxford:
1925 *February* 19.

Visual Observations of Comets 1924*b* (*Encke*), 1924*c* (*Finsler*), *and Planet* 1924*TD* (*Baade*), *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The following observations were made with the Sheepshanks equatorial (aperture 6·7 inches) by taking transits over two cross-wires, each inclined 45° to the parallel of declination.

Magnifying power, 55. Observer, A. C. D. Crommelin.

G.M.T. (Old Style) 1924.	Comet − Star. R.A.	Comet − Star. Decl.	No. of Comps.	Log Factors of Parallax.	Comet. R.A. for 1924·0.	Comet. Decl. for 1924·0.	Star
			Encke's Comet 1924*b*.				
Oct. d 12·71441	m s +2 32·40	′ ″ + 4 48·0	2	9·554*n* 0·795	h m s 11 17 41·72	° ′ ″ +13 53 11·6	*a*
			Comet 1924*c* (*Finsler*).				
Sept. 22·30322	+3 40·55	− 4 35·0	3	9·556 0·822	14 13 48·30	+ 7 24 1·4	*b*
22·30515	+0 25·80	+17 43·2	2	9·558 0·823	14 13 49·02	+ 7 23 54·1	*c*
23·30385	−0 29·94	−12 18·9	3	9·555 0·825	14 19 54·38	+ 5 57 31·9	*d*
23·30385	−1 21·43	− 0 1·4	3	9·555 0·825	14 19 54·45	+ 5 57 31·9	*e*
24·30389	+1 24·10	+ 0 45·9	4	9·552 0·828	14 25 33·31	+ 4 34 55·4	*f*
27·29650	−1 15·32	−13 23·7	5	9·544 0·835	14 40 22·49	+ 0 48 50·2	*g*
28·29394	−0 16·46	+12 6·2	7	9·542 0·837	14 44 43·25	− 0 19 49·7	*h*
29·29566	−5 12·35	+ 0 8·4	1	9·544 0·839	14 48 48·55	− 1 25 24·2	*j*
			Planet 1924*TD* (*Baade*).				
Nov. 3·43268	+1 44·39	+ 3 30·0	4	9·446 0·797	21 59 5·63	+ 8 20 10·1	*k*
10·28219	+0 52·98	+11 2·2	3	8·614*n* 0·807	22 30 36·78	+ 4 30 16·5	*l*
17·26852	−0 1·32	+ 5 52·2	7	8·881*n* 0·828	23 0 37·54	+ 1 17 11·6	*m*
17·27150	−0 56·12	− 4 6·2	6	8·841*n* 0·828	23 0 38·27	+ 1 17 8·5	*n*

The observations are corrected for the effect of the motion of the objects, also for differential refraction, but not for parallax. It will be noticed that the places are not apparent, but are for 1924·0, being obtained simply by applying the differences "object – star" to the mean places of the stars. This is a new departure as regards visual observations at Greenwich. The reasons for it are given in a paper by Mr. G. Merton, who, on p. 516, also gives the corrections for differential precession, nutation, and aberration to be applied to the above places. They are negligible, although some of the differences "comet – star" are unusually large; in these cases nearer stars were invisible, owing to low altitude or haze.

Sept. 28 and 29. Comet faint and difficult to see, owing to low altitude and twilight; observations not considered very exact.

Comparison Stars.

	Name.	Mag.	Mean Place 1924·0. R.A. h m s	Decl. ° ′ ″	Authority.
a	B.D.+14° 2374	7·0	11 15 9·32	+13 48 23·6	Bordeaux Astrog. with P.M. $-^{s}·009$, $+''·005$.
b	B.D.+7° 2760	7·8	14 10 7·75	+ 7 28 36·4	Leipzig A.G.C. and Paris with P.M. $+^{s}·003$, $-''·075$.
c	B.D.+7° 2766	8·7	14 13 23·22	+ 7 6 10·9	Leipzig A.G.C. and Kustner with P.M. $-^{s}·003$, $''·00$.
d	Piazzi XIV. 73	5·2	14 20 24·32	+ 6 9 50·8	Boss P.G.C.
e	B.D.+6° 2878	7·7	14 21 15·88	+ 5 57 33·3	Leipzig A.G.C. and Paris with P.M. $+^{s}·002$, $+''·007$
f	B.D.+4° 2866	8·5	14 24 9·21	+ 4 34 9·5	Abbadia.
g	108 Virginis	5·9	14 41 37·81	+ 1 2 13·9	Boss P.G.C.
h	B.D.−0° 2886	6·7	14 44 59·71	− 0 31 55·9	Greenwich Cat. 1925.
j	B.D.−1° 2999	8·7	14 54 0·90	− 1 25 32·6	Abbadia with P.M. $-^{s}·002$, $-''·01$.
k	B.D.+7° 4778	8·5	21 57 21·24	+ 8 16 40·1	Leipzig A.G.C.
l	B.D.+3° 4725	9·2	22 29 43·80	+ 4 19 14·3	Kustner.
m	B.D.+0° 4961	8·2	23 0 38·86	+ 1 11 19·4	Abbadia with P.M. $-^{s}·006$, $-''·10$.
n	B.D.+0° 4964	8·5	23 1 34·39	+ 1 21 14·7	Albany, Nicolajew, Kustner, Abbadia.

Photographic Observations of Comets and Minor Planets made at the Royal Observatory, Greenwich, in 1924. By G. Merton, M.A.

(*Communicated by the Astronomer Royal.*)

Through the kindness of the Astronomer Royal I was enabled to obtain some photographic observations of comets and minor planets, the results of which are given below. The two plates of Baade's Planet, obtained on November 3 and 27, were kindly taken for me by Mr. C. Davidson and Mr. P. J. Melotte respectively. The remaining

plates were taken by myself. I carried out the reductions by the usual method used at the observatory, and the plate constants in each case were determined from six or more stars for which the most modern positions were used. The positions are given in the usual astrographic form, and the last column indicates the instrument used, 30-inch reflector (R) or 13-inch astrographic (A), and the number of exposures on the plate on which the result depends. I am much indebted to Dr. A. C. D. Crommelin and Mr. C. Davidson for their help and advice, and to Professor H. H. Turner for the loan of the machine with which most of the plates were measured.

	G.M.T. (Old Style). 1924.	R.A. 1924·0.	Dec. 1924·0.	log Parx. Factors.		
		Comet 1924*a* (*Reid*).				
		h m s	° ′ ″			
1	Oct. 6·64432	8 39 55·75	+12 18 42·9	$9^{n}522$	0·789	R 1
		Encke's Comet 1924*b*.				
2	July 31·57729	3 24 5·8	+28 2 37	$9^{n}600$	0·727	R 1
3	Aug. 27·54042	5 1 14·04	34 59 29·3	$9^{n}648$	0·727	R 1
4	31·50880	5 22 33·23	35 53 27·3	$9^{n}652$	0·777	R 1
5	Oct. 6·66526	10 29 14·74	21 31 1·7	$9^{n}591$	0·786	R 2
6	12·70553	11 17 38·66	13 53 47·7	$9^{n}559$	0·798	R 1
7	12·71381	11 17 42·37	13 53 10·5	$9^{n}552$	0·794	R 1
8	12·71900	11 17 44·67	13 52 46·5	$9^{n}547$	0·791	R 1
9	19·72327	12 7 33·60	4 50 36·0	$9^{n}546$	0·825	A 1
		Comet 1924*d* (*Wolf*).				
10	Dec. 26·25553	4 7 3·51	+23 51 19·2	$9^{n}515$	0·698	R 1
11	26·40174	4 6 59·83	23 49 22·6	$8^{n}150$	0·607	R 1
		Planet (4) *Vesta*.				
12	Aug. 5·53381	22 24 56·80	−18 32 37·4	$8^{n}817$	0·915	R 2
		Planet 1924*TD* (*Baade*).				
13	Oct. 28·47776	21 30 27·23	+12 4 32·5	9·535	0·797	R 2
14	30·35275	21 39 33·75	10 52 7·6	9·125	0·761	A 2
15	Nov. 3·31383	21 58 32·11	+ 8 24 29·4	8·596	0·777	R 2
16	27·27407	23 39 23·03	− 2 1 38·0	$8^{n}794$	0·848	A 2
17	Dec. 14·26367	0 34 54·04	− 4 41 25·0	$8^{n}833$	0·862	A 2
18	26·36830	1 8 18·94	− 5 0 38·8	9·272	0·860	R 3

Notes.—1. Image faint; mag. $=14\frac{1}{2}$. Position agrees with Dr. Baade's in *B.Z.*, 1925, No. 3. 2. Image very diffuse and uncertain; position therefore doubtful. Mag. $=15$. 3. Image faint. 4. Interrupted by clouds, and therefore I had to measure to equivalent star images of the beginnings of the trails and assume that the trailing was correct. This was checked and found satisfactory. Image faint. 10. Image very faint; mag. $=15$. 11. Image faint and intersected by star trail. 12. Comparison with ephemeris from Leveau's tables by Mr. Bawtree (*B.A.A. Handbook*, 1924), O–C $-0^{s}\cdot38$, $+0''\cdot1$. 14. Mag. $=8\cdot9$. 17. Mag. $=12$. 18. Mag. $=10\frac{1}{2}$.

Royal Observatory, Greenwich:
1925 *February* 10.

The Reduction and Publication of Observations of Minor Planets and Comets. By G. Merton, M.A.

The observed places of minor planets and comets are now frequently published in the form in which they are referred to the mean equinox of the beginning of the year. It is suggested that this practice might well become universal. This was urged by Ristenpart in 1902 and by several others since then.* Unfortunately most of those responsible for the observations seem slow to appreciate this suggestion, which would save them work, and fail to realise that the places they publish as "apparent" are not in the form most convenient to the orbit computer, who usually takes out the "Reduction to Apparent" which they have calculated, and makes no further use of it. Thus the computer uses, as a rule, the mean place obtained by adding the observed differences in Right Ascension ($\Delta\alpha$) and Declination ($\Delta\delta$) direct to the mean place of the comparison star (*vide seq.*).

This note is intended, firstly, to draw attention once again to this point with a view to soliciting discussion before such matters are dealt with by the committee concerned at the International Astronomical Union meeting at Cambridge in July next. Secondly, to suggest that it is very desirable, in order to avoid the mistakes which are sometimes made, that the terms "Astrographic," "Mean Place," and "True Place," when applied to observations of minor planets and comets, should be clearly defined. Thirdly, to suggest that the form in which the observations should be published should now be officially indicated, so that the material shall in future be on a uniform basis suited as far as possible to the convenience of orbit computers, by whom the observations are used. Certain points involved will now be discussed under their separate headings, and the recommendations given at the end.

Mean Places.—When the observed positions of comets (or planets) referred to the mean equinox are given, they are usually printed under, for example, R.A. 1924·0, Dec. 1924·0, or they are specified as "Astrographic Places," † or less commonly as "Mean Places" or "Apparent Mean Places." *All these have been used with the same meaning.*

The expression "Mean Place," when referring to a moving body such as a comet or planet, is not, perhaps, as fortunate a one as could be wished for, and is, for that matter, not often found in print. Nevertheless its use would be very convenient if its definition were officially laid down, for it is shorter than the word "astrographic," which, moreover, has come naturally to be applied to photographic observations, or than the word "astrovisual," which might conveniently be coined to refer in a similar way to visual observations. An actual definition of mean place in this connection is rarely to be found in standard

* F. Ristenpart, *A.N.*, **160** (3832–33); E. C. Bower, *A.J.*, **32**, 48; E. Millosevich, *ibid.*, p. 93; *Union Observatory Circulars*, Nos. 11 and 60 (pp. 225, 226), but note that the definition of mean place there given is incorrect (*vide seq.*).

† *Greenwich Observations*, volume for 1910, p. C 84, last para. but one. Previous to this they were indicated as "apparent mean places."

works, but Frischauf, in his *Grundriss der theoretischen Astronomie*, § 31, gives it.* The expression occurs occasionally in works dealing with the computation of orbits, and reference to these † leaves no doubt that, as Frischauf indicates, such positions are referred directly to the mean places of the stars *without* the application of any corrections for aberration, and are dated for the actual times of the observations.

Mean places are usually obtained, in the case of visual observations, by adding the measured differences in Right Ascension ($\Delta\alpha$) and Declination ($\Delta\delta$), corrected for differential refraction, directly to the mean place of the stars. They then differ from the photographic places only by the amount of the *differential* precession, nutation, and star aberration, which, in photographic reductions, are automatically taken account of in the plate constants. These corrections, which in visual observations are usually quite negligible, will be discussed later in this paper.

Misunderstandings concerning the recognised meaning of "mean" and "astrographic" places are not lacking. The *Union Observatory Circulars*, **34**, 267, and **60**, 225, give a definition of "mean" place which includes aberration corrections, and is, in fact, really the definition of a "true mean place" as given below. Again, on page 658 of *Popular Astronomy* for 1924 December we find, in a list of observations of Planet 1924TD (Baade), that "mean" and "astrographic" places have been specifically differentiated between, presumably because the observations when received for publication were described in these terms respectively.

True and True Mean Places.—The true place of a star indicates the true direction (neglecting the effect of solar motion) from which the light is coming, *i.e.* the true direction as seen from the sun; but the true place of a comet (or planet) is the direction of its actual position in space, which is another matter. The true place is essentially an ephemeris place (α vera, δ vera), and is defined as that place which results from the combination of the actual geometrical place of the comet (or planet) with the earth's co-ordinates at the *same instant*, both being referred to the equator (or ecliptic) of the true equinox. ‡

Where the equinox used is the mean equinox of the beginning of the year, it is suggested, for want of a better name, that the place should be called a "True Mean Place." The ephemerides of comets and minor planets are now usually published in the form of true mean places, hence the desirability of having a name for them.

An observation of a comet in the form of a mean place is then reduced to the true mean place, for direct comparison with such an

* J. Frischauf, *Grundriss der theoretischen Astronomie*, 3rd ed. (Leipzig, 1922), p. 73.

† R. v. Oppolzer, *Bahnbestimmung*, vol. i. (2nd ed., 1882), pp. 312 and 382; or the French edition by Pasquier (Paris, 1886), pp. 320 and 391; A. O. Leuschner, *L.O. Pub.*, **7**, 26, line 4, also p. 312; E. C. Bower, *A.J.*, **32**, 48.

‡ Bauschinger, *Bahnbestimmung* (Leipzig, 1903), chap. vi., especially pp. 85 and 86; Chauvenet, *Spherical and Pract. Astron.* (5th ed.), vol. i., § 361 (p. 603) and § 396 (p. 642), but the method § 412 (p. 657) is *not* the one usually used. Regarding ephemerides of true places, see Oppolzer, *Bahnbestimmung*, vol. i. (2nd ed.), p. 251, also p. 326; French ed., pp. 256 and 334.

ephemeris, by applying to the observed place the parallax corrections and *adding* the star aberration,* namely,

$$+\tfrac{1}{1\cdot 5}h \sin (\mathrm{H}+\alpha) \sec \delta \quad \text{in R.A.},$$

and

$$+h \cos (\mathrm{H}+\alpha) \sin \delta + i \cos \delta \quad \text{in Dec.}$$

and then *subtracting* the light time from the observation date.†

Corrections for Differential Precession, Nutation, and Aberration.—These corrections are applicable to the measured difference of position between the star and the comet (or planet). In photographic results they are automatically taken into account in the plate constants, and need not be further mentioned here.‡ In the case of a visual observation, since the corrections rarely exceed half a second of arc of a great circle, they may usually be considered negligible compared to the uncertainties of the observation itself and of the place of the comparison star used, which together are not usually less than one second of arc in the case of planets, and often exceed five seconds in the case of comets.

It is clear, therefore, that the errors due to the neglect of these differential corrections may be treated as accidental, and neglected.§ Exception might perhaps be made in cases where the distance between the object and the comparison star is very large, say of the order of $1\frac{1}{2}^\circ$ or more.

In any case, the differential corrections are certainly no more labour to compute than the whole corrections. The formulæ were gone into by Ristenpart (*A.N.*, 3832–33), but the arrangement which he adopted, and which required subsidiary annual tables, was perhaps not the most convenient. The differential formulæ without modification can be worked with ease, using a slide-rule or three-figure tables.

It seems, however, very desirable that the observatory from which the observation originates should *not* do this work, but should publish the observation corrected only for differential refraction where necessary, give the apparent $\Delta\alpha$ and $\Delta\delta$, and leave the remaining differential corrections to the orbit computer, who can calculate and apply them in whatever way he prefers. The observatory of origin being thus relieved of this work, is surely not in a position to object to the proposed change !

The formulæ for these corrections will now be given. Neglecting second-order terms and differentiating the formulæ used by the usual ephemerides, ||

$$\alpha(\mathrm{app}) = \alpha_0 + f + g \sin (\mathrm{G}+\alpha) \tan \delta + \mathrm{h} \sin (\mathrm{H}+\alpha) \sec \delta,$$
$$\delta(\mathrm{app}) = \delta_0 + g \cos (\mathrm{G}+\alpha) + h \cos (\mathrm{H}+\alpha) \sin \delta + i \cos \delta,$$

* Or subtracting the precession and nutation from the apparent place, if that is given : see H. Andoyer, *Cours d'Astronomie* (3rd ed., Paris, 1923), vol. i. chap. xii. p. 261; H. Andoyer, *Mécanique Céleste* (Paris, 1923), vol. i. pp. 143–144; Oppolzer, *Bahnbestimmung*, vol. i. (2nd ed.), p. 338; French ed., p. 347.

† *Greenwich Observations*, volume for 1910, p. C 84, last para. but one.

‡ See, for example, H. H. Turner, *M.N.*, **54**, 14.

§ F. Ristenpart, *A.N.*, **160** (3832–33), 289; *Union Observatory Circular*, No. 11, pp. 84 and 86.

|| The *American Ephemeris*, *Connaissance des Temps*, and the *Berliner Astronomisches Jahrbuch* are slightly more convenient than the *Nautical Almanac*, as they give G and H expressed in time.

we obtain, after a simple rearrangement, formulæ giving the differential corrections for precession, nutation, and star aberration to be added algebraically to the apparent $\Delta\alpha$ and $\Delta\delta$, measured in the sense "object *minus* star." These formulæ, (1) and (2), together with tables for facilitating the calculation, are given on the opposite page.

Example 1.—Suppose, to take an exaggerated case, we are given the position of δ Scorpii referred to β_1 Scorpii for 1926 May 21·0, namely, $\Delta\alpha = -5^m\ 10^s{\cdot}554$, $\Delta\delta = -168'\ 29''{\cdot}94$, which are the apparent differences taken from the *Nautical Almanac*. Then for the corrections to these :

α (mean)	$15^h\ 59^m$	δ (mean)	$-21°{\cdot}0$	$\log g$	0·51
G	$5^h\ 8^m$	G+α	$21^h\ 7^m$	$\log h$	1·30
H	$13^h\ 55^m$	H+α	$5^h\ 54^m$	i	$-4''{\cdot}2$

$$A = -7\times{\cdot}36+1 = -2$$
$$B = -7-58\times{\cdot}36 = -28 \qquad \tfrac{1}{15}\sec\delta\ 0{\cdot}07$$
$$D = +13\times{\cdot}36-1\times{\cdot}93 = +4 \qquad \sec\delta\ 1{\cdot}07$$

$$\partial\Delta\alpha\,.\cos\delta\,.\,10^4 = -2\times5{\cdot}2-2{\cdot}0\times168 = -346 \qquad \partial\Delta\alpha\,.\,10^4 = -370$$
$$\partial\Delta\delta\,.\,10^4 = +420\times5{\cdot}2-4\times168 = +1512$$

Thus the corrections to be added to the given $\Delta\alpha$'s and $\Delta\delta$'s are $-0^s{\cdot}037$ and $+0''{\cdot}15$. A three-figure computation gave $-0^s{\cdot}036$, $+0''{\cdot}16$, and the *Nautical Almanac* places $-0^s{\cdot}035$, $+0''{\cdot}16$.

Example 2.—The observation of Comet 1924*c* (Finsler), made by Dr. A. C. D. Crommelin at Greenwich on 1924 September 29·3 (see p. 510).

α	$14^h\ 52^m$	δ	$-1°{\cdot}4$	$\log g$	1·09	$\Delta\alpha$	$-5^m{\cdot}2$
G	$2^h\ 10^m$	G+α	$17^h\ 2^m$	$\log h$	1·27	$\Delta\delta$	$+0'{\cdot}1$
H	$5^h\ 32^m$	H+α	$20^h\ 24^m$	i	$+8''{\cdot}1$		

$$A = +9\times{\cdot}03+32 = +32$$
$$B = -34+44\times{\cdot}03 = -33 \qquad \tfrac{1}{15}\sec\delta\ 0{\cdot}07$$
$$D = -23\times{\cdot}03-32\times1{\cdot}0 = -33 \qquad \sec\delta\ 1{\cdot}00$$

$$\partial\Delta\alpha\,.\cos\delta\,.\,10^4 = +32\times5{\cdot}2+2\times0{\cdot}1 = +166 \qquad \text{hence } \partial\Delta\alpha = +0^s{\cdot}02$$
$$\partial\Delta\delta\,.\,10^4 = +495\times5{\cdot}2-33\times0{\cdot}1 = +2571 \qquad \partial\Delta\delta = +0''{\cdot}3$$

To illustrate further how negligible these corrections are as a rule, the corrections to all the observations on p. 510 were calculated. The following are the results :

Comet 1924*c* (Finsler).

	s	″
Sept. 22	−0·01	−0·2
22	·00	− ·1
23	·00	+ ·1
23	·00	+ ·1
24	·00	− ·1
27	·00	+ ·1
28	·00	·0
29	+0·02	+0·3

Encke's Comet 1924*b*.

	s	″
Oct. 12	+0·01	−0·1

Planet 1924TD (Baade).

	s	″
Nov. 3	−0·01	0·0
10	·00	·0
17	·00	·0
17	0·00	0·0

Table I.

Giving in units of the fourth decimal place :

S=g sin 1′ sin (G+α) and C=g sin 1′ cos (G+α) with arguments G+α and log g,
or S=h sin 1′ sin (H+α) and C=h sin 1′ cos (H+α) ,, ,, H+α and log h.

log. g / log. h	0·00.		0·50.		0·80.		0·90.		1·00.		1·10.		
G+α / H+α	S.	C.	S.	C.	S.	C.	S.	C.	S.	C.	S.	C.	
h m													h m
0 0	0	3	0	9	0	18	0	23	0	29	0	37	6 0
20	0	3	1	9	2	18	2	23	3	29	3	36	5 40
0 40	1	3	2	9	3	18	4	23	5	29	6	36	20
1 0	1	3	2	9	5	18	6	22	8	28	9	35	5 0
20	1	3	3	9	6	17	8	22	10	27	13	34	4 40
1 40	1	3	4	8	8	17	10	21	12	26	15	33	20
2 0	1	3	5	8	9	16	12	20	15	25	18	32	4 0
20	2	2	5	8	11	15	13	19	17	24	21	30	3 40
2 40	2	2	6	7	12	14	15	18	19	22	24	28	20
3 0	2	2	7	7	13	13	16	16	21	21	26	26	3 0

h m	1·10.		1·20.		1·30.		1·35.		1·40.		1·45.		h m
0 0	0	37	0	46	0	58	0	65	0	73	0	82	6 0
10	2	37	2	46	3	58	3	65	3	73	4	82	5 50
20	3	36	4	46	5	58	6	65	6	73	7	82	40
30	5	36	6	46	8	58	8	65	10	72	11	81	30
40	6	36	8	45	10	57	11	64	13	72	14	81	20
0 50	8	36	10	45	13	57	14	64	16	71	18	80	10
1 0	9	35	12	45	15	56	17	63	19	71	21	79	5 0
10	11	35	14	44	17	55	20	62	22	70	25	78	4 50
20	13	34	16	43	20	54	22	61	25	69	28	77	40
30	14	34	18	43	22	54	25	60	28	68	31	76	30
40	15	33	19	42	25	53	28	59	31	66	35	74	20
1 50	17	33	21	41	27	51	30	58	34	65	38	73	10
2 0	18	32	23	40	29	50	33	56	37	63	41	71	4 0
10	20	31	25	39	31	49	35	55	39	62	44	69	3 50
20	21	30	26	38	33	48	37	53	42	60	47	67	40
30	22	29	28	37	35	46	40	52	45	58	50	65	30
40	24	28	30	35	37	44	42	50	47	56	53	63	20
2 50	25	27	31	34	39	43	44	48	49	54	55	60	10
3 0	26	26	33	33	41	41	46	46	52	52	58	58	3 0
	C.	S.	C.	S.	C.	S.	C.	S.	C.	S.	C.	S.	

Table II.

i.	log. i.	i sin 1′ / 10^4.
0	−∞	0
1	0·00	3
2	·30	6
3	·48	9
4	·60	12
5	·70	15
6	·78	17
7	·85	20
8	·90	23
9	·95	26

Table III.

δ.	sin δ.	cos δ.	$\frac{1}{15}$ sec δ.	sec δ.	δ.	sin δ.	cos δ.	$\frac{1}{15}$ sec δ.	sec δ.
°					°				
0	0·00	1·00	0·07	1·00	45	0·71	0·71	0·09	1·41
5	·09	1·00	·07	1·00	50	·77	·64	·10	1·56
10	·17	·98	·07	1·02	55	·82	·57	·12	1·74
15	·26	·97	·07	1·04	60	·87	·50	·13	2·00
20	·34	·94	·07	1·06	65	·91	·42	·16	2·37
25	·42	·91	·07	1·10	70	·94	·34	·19	2·92
30	·50	·87	·08	1·16	75	·97	·26	·26	3·86
35	·57	·82	·08	1·22	80	·98	·17	·38	5·76
40	·64	·77	·09	1·31	85	1·00	·09	·76	11·47
45	·71	·71	·09	1·41	90	1·00	0·00	∞	∞

Let

$$\left.\begin{aligned} A &= g \sin 1' \cos(G+\alpha) \sin\delta + h \sin 1' \cos(H+\alpha) \\ B &= g \sin 1' \sin(G+\alpha) \quad\quad + h \sin 1' \sin(H+\alpha) \sin\delta \\ D &= i \sin 1' \sin\delta \quad\quad - h \sin 1' \cos(H+\alpha) \cos\delta \end{aligned}\right\} \quad . \quad . \quad (1)$$

then the corrections (in seconds of time and arc respectively) to be *added* to the observed $\Delta\alpha$ and $\Delta\delta$ are given by :

$$\left.\begin{aligned} \partial\Delta\alpha \,.\, \cos\delta &= -A \,.\, \Delta\alpha^m - B\tfrac{1}{15}\sec\delta \,.\, \Delta\delta' \\ \partial\Delta\delta &= +15B \,.\, \Delta\alpha^m + D \,.\, \Delta\delta' \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

where $\Delta\alpha^m$ and $\Delta\delta'$ are in minutes of time and arc respectively.

Differential Refraction.—The corrections which have to be applied on this account will depend upon the micrometer used, whether its orientation was checked on the meridian or in an observational field, and how the observations were made. These corrections, together with those for the proper motion of the body during the observations and for the errors of the instrument, clearly depend upon circumstances which are only fully known at the observatory of origin, which therefore should be responsible, as hitherto, for making them.* The printed $\Delta\alpha$ and $\Delta\delta$ are understood to have had them applied *before* publication.

It must not be overlooked that the corrections for differential refraction are often much larger than the other differential corrections, and, for observations made at a low altitude, may amount to several seconds of arc.

Aberration Corrections.—In the computation of orbits it is necessary to take account of the effect of the aberration due to the earth's motion and that due to the motion of the planet or comet (light time).† When the observed positions have been corrected in the manner already suggested, the task of making the additional corrections for the parallax, aberration, and light time is essentially the orbit computer's, and it should undoubtedly be left to him.

Sometimes the star-aberration corrections in R.A. and Dec. are published with the observations. There seems to be no good reason why this should be done. The computer of a preliminary orbit uses, as a rule, a method which allows fully for the aberration effects but does not require the use of these star-aberration corrections, while the computer of a definitive orbit will in any case compute these corrections for himself for the whole period covered by the observations. Consequently, the publication of a few odd corrections of this kind is of little use, and should be discontinued.

Computational Errors.—In urging the universal use of astrographic or astrovisual places for comets and minor planets, it may be pointed out that, apart from the fact that the computation of the "Reduction to Apparent" is not needed, the computation of it is also a source of error not to be disregarded. Errors from this and other causes are distressingly frequent in the reduction of observations of this kind. The fact that such observations wrongly reduced are not only useless, but also a great nuisance to the orbit computer who, in the case of a first orbit, may waste precious hours of work on their account, is not fully appreciated.

There seems little doubt that the computations of the reductions of observations frequently go unchecked. An examination of the results in the determinations of definitive orbits shows that about *one observation in every fifteen has to be rejected* either wholly or in part for errors

* Chauvenet, *Spherical and Pract. Astron.*, vol. ii. chap. x., "Micrometer Observations"; Valentiner's *Handwörterbuch der Astron.*, **3***a*, 64, article by E. Becker, "Mikrometer und Mikrometer Messungen"; H. Andoyer and A. Lambert, *Cours d'Astron.* (2nd ed., Paris, 1924), **2**, 161. As regards photographic results, see for example H. H. Turner, *M.N.*, **54**, 13 and 14.

† See references given on p. 514 (‡), and on p. 515 (*).

which considerably exceed the observational uncertainty.* It seems reasonable to suppose that smaller undetected errors will be present, and will tend to give unnecessarily large probable errors in the results.

From every point of view, therefore, it seems highly desirable that the work of applying corrections to the observations should be reduced to a minimum at the observatory of origin, which should also recognise its responsibility for having such computations as are necessary *adequately* checked before publication. The results should be printed or telegraphed on a uniform basis. For this purpose the " mean " place as here defined has already been found in practice to be quite suitable, and should, for reasons already discussed, be universally adopted.

Recommendations.

1*a*. That all published positions of minor planets and comets shall give the Greenwich date and time of observation and the *mean* place, as herein defined, of the body.

1*b*. That this shall also apply to astronomical telegrams, the name of the observatory of origin being included as the whole, or a part of, the signature.

2*a*. That *Mean Places* shall be the places obtained by adding the observed differences in Right Ascension ($\Delta\alpha$) and Declination ($\Delta\delta$), already corrected for differential refraction, directly to the mean places of the comparison stars ; and that the equinox used shall be the mean equinox of the beginning of the year unless otherwise stated. The dates shall give the actual times of observation (*i.e.* uncorrected for light time).

2*b*. That in the case of visual observations it shall be understood that the published $\Delta\alpha$ and $\Delta\delta$ and the resulting Right Ascension and Declination have *not* been corrected for the differential effects of precession, nutation, and aberration, unless the contrary is specifically stated.

2*c*. That in the case of published photographic observations (in which $\Delta\alpha$ and $\Delta\delta$ will not usually be given) it will be assumed that all the differential corrections have been included.

2*d*. That the terms " astrographic " and " astrovisual " may conveniently be applied to photographic and visual mean places respectively.

3*a*. That a *True Place* shall be the geometrical position (*i.e.* not affected by aberration) referred to the *true* equinox of date.

3*b*. That a *True Mean Place* shall be the geometrical position referred to the mean equinox, and that this mean equinox shall be for the beginning of the year, unless otherwise stated. That is to say, it shall be the observed mean place properly corrected for aberration, light time, and parallax, for direct comparison with the usual form of ephemeris obtained from the combination of the actual positions of the comet (or planet) and the earth at the *same* instant, both referred to the mean equinox of the beginning of the year.

4. That the computation and application of the aberration and

* See also Luc Picart, *Calcul des Orbites et des Éphémérides* (Paris, 1913). He quotes Faye, p. 164.

parallax corrections shall always be left entirely to the orbit computer; and that it is also desirable that the differential corrections for precession, nutation, and star aberration, but *not* refraction, should also be left to him.

5. That, as hitherto, the observatory of origin shall be responsible for making the corrections to $\Delta\alpha$ and $\Delta\delta$ for differential refraction before publication, and for computing and publishing the parallax factors.

The Royal Observatory, Greenwich:
1925 *March* 12.

Occultations of Stars by the Moon, observed at the Astronomical Observatory, Kiew, in the Year 1924, *and Micrometer Observations of the Position of Mars.* By Professor S. D. Tscherny.

No.	Date 1924.	G.M.T. h m s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.
1	Apr. 8	4 55 14·47	θ^1 Tauri	4·2	D	R 145	T
2	8	4 55 13·76	θ^1 Tauri	4·2	D	K 24	D
3	8	4 55 14·85	θ^1 Tauri	4·2	D	G 58	K
4	8	5 48 26·22	θ^1 Tauri	4·2	R	R 145	T
5	8	5 57 36·71	264 B Tauri	4·8	D	R 145	T
6	8	5 57 36·66	264 B Tauri	4·8	D	K 24	D
7	8	5 57 36·50	264 B Tauri	4·8	D	G 58	K
8	8	6 4 21·76	Anonyma	6·0	D	K 24	D
9	8	7 2 7·10	264 B Tauri	4·8	R	R 145	T
10	8	7 35 32·61	275 B Tauri	6·5	D	R 145	T
11	8	7 35 33·26	275 B Tauri	6·5	D	K 24	D
12	8	7 35 32·44	275 B Tauri	6·5	D	G 58	K
13	May 9	7 22 43·30	*f* Geminorum	5·3	D	R 145	T
14	9	7 22 43·44	*f* Geminorum	5·3	D	K 24	D
15	9	7 22 43·22	*f* Geminorum	5·3	D	G 58	K
16	9	8 15 26·14	*f* Geminorum	5·3	R	R 145	T
17	18	6 32 10·82	η Libræ	5·5	D	R 145	T
18	18	7 0 6·24	η Libræ	5·5	R	R 145	T
19	June 12	6 48 57·28	65 Virginis	6·0	D	R 145	T
20	12	6 48 57·18	65 Virginis	6·0	D	G 87	K
21	12	6 48 57·41	65 Virginis	6·0	D	B 61	L
22	12	7 21 6·49	65 Virginis	6·0	R	R 145	T
23	12	7 25 36·56	66 Virginis	5·7	D	G 87	K
24	12	7 25 37·33	66 Virginis	5·7	D	B 61	L
25	12	8 24 28·08	66 Virginis	5·7	R	R 145	T

No.	Date 1924.	G.M.T. h m s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.
26	July 6	6 17 15·05	ρ Leonis	3·8	R	R 145	T
27	Aug. 6	7 40 51·61	88 Virginis	6·5	D	R 145	T
28	8	6 15 24·66	γ Libræ	4·0	D	R 145	T
29	8	6 15 24·46	γ Libræ	4·0	D	K 24	D
30	8	6 15 24·21	γ Libræ	4·0	D	G 58	K
31	8	6 15 24·73	γ Libræ	4·0	D	B 61	L
32	8	7 25 27·21	γ Libræ	4·0	R	R 145	T
33	22	10 34 42·46	71 Tauri	4·6	D	R 145	T
34	22	10 37 7·07	70 Tauri	6·4	D	R 145	T
35	22	11 3 41·71	70 Tauri	6·4	R	R 145	T
36	22	11 3 41·48	70 Tauri	6·4	R	G 58	K
37	22	11 3 42·18	70 Tauri	6·4	R	R–H 89	P
38	22	11 3 42·28	70 Tauri	6·4	R	U 74	Ch
39	22	11 27 3·88	71 Tauri	4·6	R	R 145	T
40	22	11 27 2·62	71 Tauri	4·6	R	G 58	K
41	22	11 27 2·66	71 Tauri	4·6	R	R–H 89	P
42	22	11 27 2·86	71 Tauri	4·6	R	U 74	Ch
43	22	11 47 10·58	θ^2 Tauri	3·6	D	R 145	T
44	22	11 47 9·07	θ^2 Tauri	3·6	D	R–H 89	P
45	22	11 47 9·47	θ^2 Tauri	3·6	D	U 74	Ch
46	22	12 55 5·44	θ^2 Tauri	3·6	R	R 145	T
47	22	12 55 5·36	θ^2 Tauri	3·6	R	R–H 89	P
48	22	12 55 4·86	θ^2 Tauri	3·6	R	U 74	Ch
49	22	12 56 4·28	θ^1 Tauri	4·2	R	R 145	T
50	22	12 56 2·98	θ^1 Tauri	4·2	R	G 58	K
51	22	12 56 3·02	θ^1 Tauri	4·2	R	R–H 89	P
52	22	12 56 3·62	θ^1 Tauri	4·2	R	U 74	Ch
53	22	13 5 42·70	264 B Tauri	4·8	D	R 145	T
54	22	13 5 39·61	264 B Tauri	4·8	D	R–H 89	P
55	22	13 5 40·79	264 B Tauri	4·8	D	U 74	Ch
56	22	13 7 23·43	Anonyma	6·4	D	R 145	T
57	22	13 7 20·57	Anonyma	6·4	D	R–H 89	P
58	22	14 11 13·96	264 B Tauri	4·8	R	R 145	T
59	22	14 11 14·07	264 B Tauri	4·8	R	R–H 89	P
60	Sept. 16	8 58 7·68	ξ^2 Ceti	4·3	R	R 145	T
61	16	8 58 8·63	ξ^2 Ceti	4·3	R	B 61	L
62	Oct. 3	5 8 11·32	B.D. −18° 4379	8·5	D	R 145	T
63	3	5 21 32·14	29 Ophiuchi	6·4	D	R 145	T
64	3	5 21 32·41	29 Ophiuchi	6·4	D	K 24	D
65	Dec. 7	5 51 55·88	ξ^2 Ceti	4·3	D	G 58	T

Instruments used :—

R=243-mm. Merz-Repsold refractor.
K=120-mm. Utzschneider and Fraunhofer comet-seeker.
R-H=107-mm. Reinfelder and Hertel refractor.
B=81-mm. Busch refractor.
U=72-mm. Utzschneider and Fraunhofer refractor.
G=70-mm. Goerz refractor.

Observers :—

T=S. Tscherny.
D=M. Ditschenko.
P=N. Pisarenko.
Ch=L. Chajnowski.
L=E. Laurentjewa.
K=A. Karamischew.

Notes.

Nos. 5, 10, 16, 19, 28, 35, 39, 46, 49, 51, 58, 59, 60, 63 : instantaneous.
Nos. 7, 13, 15, 27, 30, 31, 32, 33, 37, 38, 41, 43, 47, 48, 52, 53, 55, 62 : good.
Nos. 1, 4, 34, 45, 54, 56 : satisfactory.
Nos. 20, 21, 22, 23, 24, 25, 65 : observed through clouds.
Nos. 18, 26 : retardation.
No. 44 : doubtful.

On 1924 December 5 the position of the centre of Mars relative to the star B.D.−4° 5913 was observed with the filar micrometer.

The observations were reduced as in *M.N.*, **84**, 772, and gave the apparent geocentric position of Mars for $4^h\ 35^m\ 40^s{\cdot}35$ G.M.T. as

$$23^h\ 31^m\ 36^s{\cdot}86,\quad -3^\circ\ 51'\ 51''{\cdot}6,$$

the position of the star being assumed to be

$$23^h\ 31^m\ 34^s{\cdot}00,\quad -3^\circ\ 51'\ 19''{\cdot}7.$$

This gives corrections to Le Verrier's tables (*C.T.*) as $-0^s{\cdot}33$, $-0''{\cdot}8$, and to Newcomb's tables with Ross's corrections (*N.A.*) $-0^s{\cdot}18$, $-0''{\cdot}3$.

Astronomical Observatory, Kiew :
1924 *February* 17.

Observations made at the Radcliffe Observatory, Oxford, of Occultations of Stars by the Moon, and of the Partial Eclipse of the Sun on 1925 January 24.

(*Communicated by the Radcliffe Observer.*)

Occultations.—Four occultations of Aldebaran have been observed here recently. The results of the first of these (1923 January 27) have already been published (*M.N.*, **83**, 480). The observation of 75 Tauri was made on the night of another occultation of Aldebaran, but cloud prevented observation of the latter.

The following observed times are given in G.M.T., the day starting at midnight in 1925. The times depend on transit observations in 1923 and 1924 and on the Paris wireless signals in 1925.

Star.	Date.	Disappearance. h m s	Reappearance. h m s	Observer.
Aldebaran	1923 Oct. 27	11 17 51·3 (*a*)	11 51 49·3 (*b*)	B.
,,	1924 Apr. 8	8 46 26·6 (*c*)	9 32 6·2±1^{s}·5 (*d*)	R.
		8 46 26·5 (*e*)	9 32 6·4 (*f*)	HB.
		8 46 26·8 (*g*)		B.
,,	1925 Jan. 6		16 4 4·4 (*h*)	HB
			16 4 4·1 (*i*)	H.
75 Tauri	1925 Feb. 2	18 55 54·6		B.

(*a*) Sky quite clear.
(*b*) Thin film of cloud gradually spread over the sky just before reappearance (10-inch : power 90).
(*c*) Naked-eye observation : disappeared instantaneously at earth-lit limb : good.
(*d*) Sudden appearance, but chronometer inaudible owing to trains (3¼-inch : power 54).
(*e*) Naked-eye observation ; instantaneous.
(*f*) Rather doubtful (1·8-inch : power 8).
(*g*) Star's image good, but Moon's limb boiling a little (10-inch : power 90).
(*h*) Doubtful observation (3¼-inch : power 54).
(*i*) Observation not very good : Moon's limb rather unsteady (10-inch : power 90).

Partial Eclipse of the Sun, 1925 January 24.—Observations of the first contact were made with the 10-inch Barclay, power 90, observer R, and with the 3¼-inch Marlborough, power 54, observer HB. Considering the low altitude of the Sun, the conditions were good at first contact, but cloud hid the Sun about a quarter of an hour afterwards. There was thin cloud over most of the sky, and no stars were seen.

The times are given in G.M.T., and depend on the Paris time-signals.

	h	m	s	Observer.
First contact	14	50	26·4	R.
	14	50	25·8	HB.

Observers: R—W. H. Robinson. HB—H. G. S. Barrett. B—J. G. Balk.

Radcliffe Observatory, Oxford :
1925 *March* 2.

Occultation of 26 Ceti at Lahore, Punjab, on 1925 January 29.

This star is double, the components being of magnitudes $6^{m}\cdot6$, 9^{m}, separation 16″. The times are given in G.M.T. (reckoned from midnight).

Star.	Disappearance (Dark Limb).	Reappearance (Bright Limb).
	h m s	h m s
B	14 22 17	..
A	14 23 5	15 21 16

Time of reappearance doubtful.
Instrument :—7-inch equatorial (Grubb).
Observer :—P. Samuels-Lall, Punjab University Observatory.
Position of Observatory :—Latitude 31° 34′, Longitude 4^{h} 57^{m} 20^{s}.